

PROBLEM 1. How many poker hands (5 cards) from a regular deck (52 cards) have at least one card from each of the four standard suits?

Hint: Let $N_{\spadesuit}$ be the collection of hands containing no spades, and similarly define $N_{\clubsuit}$, $N_{\heartsuit}$, and $N_{\diamondsuit}$. What is the relationship between the answer to this question and $|N_{\spadesuit} \cup N_{\clubsuit} \cup N_{\heartsuit} \cup N_{\diamondsuit}|$?

PROBLEM 2. Recall that D_m denote the number of derangements of $[m]$. How many derangements π of $[n]$ have $\pi(1) = 2$ and $\pi(2) = 1$? Fix some k such that $2 \leq k \leq n$; how many derangements π of $[n]$ have $\pi(1) = k$ and $\pi(k) = 1$?

PROBLEM 3. How many derangements π of $[n]$ have $\pi(1) = 2$ and $\pi(2) \neq 1$? Fix some k such that $2 \leq k \leq n$; how many derangements π of $[n]$ have $\pi(1) = k$ and $\pi(k) \neq 1$?

PROBLEM 4. Let D_n be the number of derangements of $[n]$. Use your answers to Problems 2 and 3 to find a formula for D_n in terms of D_{n-1} and D_{n-2} . Determine D_1 and D_2 by hand and then use your formula to determine D_n for $n = 3, 4, 5$, and 6 ; check that your answers match with the closed formula given in the text.