

PROBLEM 1. A *complete graph* on n vertices, denoted K_n , has every possible edge. Draw pictures of K_3 , K_4 , and K_5 . How many edges are there in a complete graph on n vertices? For a general graph $G = (V, E)$, make an inequality relating $|V|$ and $|E|$.

PROBLEM 2. A graph $G = (V, E)$ is called *bipartite* if it is possible to partition V with nonempty sets as $V = A \amalg B$ such edges only go between A and B . The *complete bipartite graph* on $p + q$ vertices, denoted $K_{p,q}$, has $|A| = p$, $|B| = q$, and all possible edges between A and B .

- (i) Draw pictures of $K_{2,3}$ and $K_{3,5}$.
- (ii) How many edges are in $K_{p,q}$?
- (iii) If $|A| = p$ and $|B| = q$ with $A \cap B = \emptyset$, how many (not necessarily complete) bipartite graphs have vertex set $A \cup B$ with $A \amalg B$ as the specified partition?

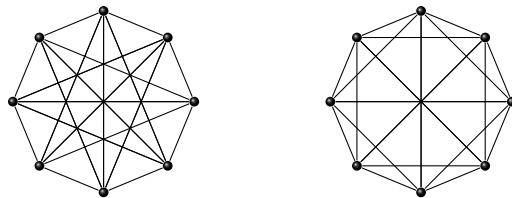
PROBLEM 3. The definition of graph isomorphism implies that isomorphic graphs have the same number of vertices and same number of edges.

- (i) Must two graphs with the same number of vertices and same number of edges be isomorphic? Prove it or find a counterexample?
- (ii) The *degree sequence* of a graph is a list of its vertex degrees in non-decreasing order. Prove that graphs with the same degree sequence have the same number of edges.
- (iii) Must two graphs with the same degree sequences be isomorphic? Prove it or find a counterexample.

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

- (i) Determine whether the following graphs are isomorphic.



- (ii) Determine whether the graphs in any pair of the following are isomorphic.

