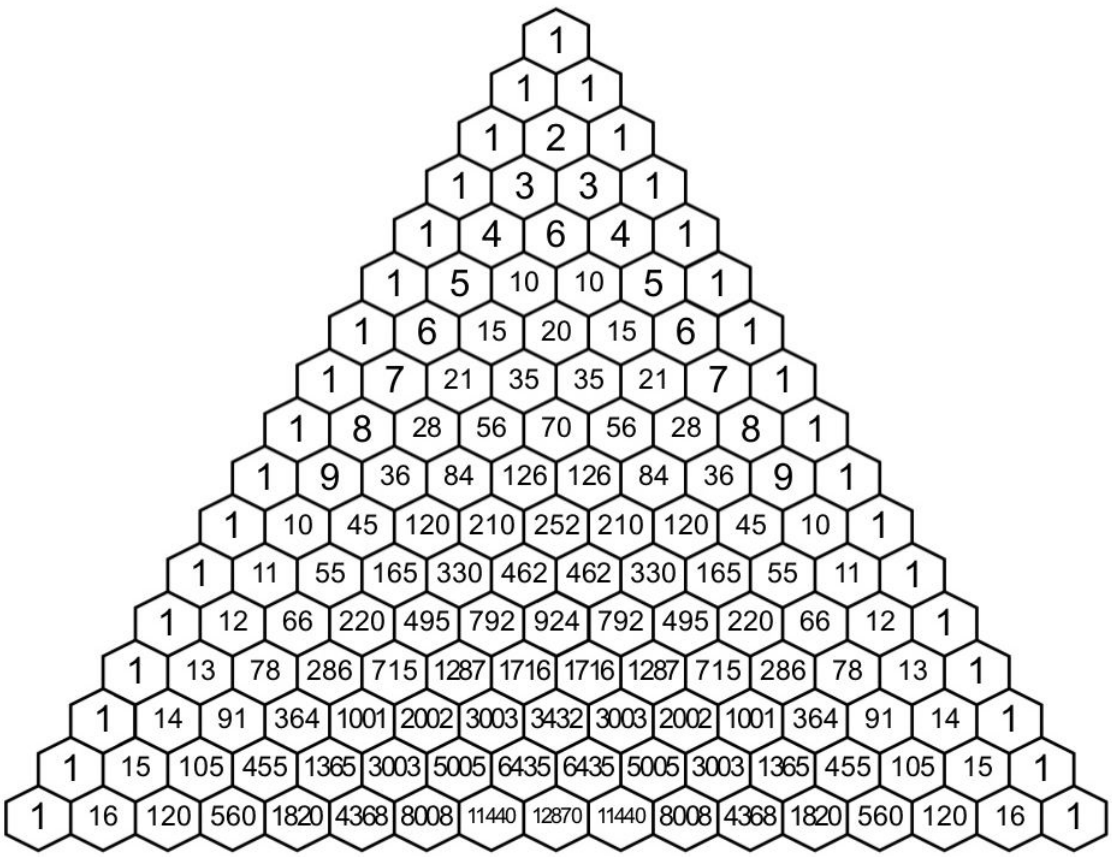
EXPLORATION 1. Color this copy of Pascal's triangle so that each odd entry is shaded. Find and prove any patterns that you observe. For instance,

(i) which rows are completely shaded?
(ii) how many entries are shaded in the n-th row?

(Remember to use the convention that the $\binom{n}{k}$ row is the n-th row.)



EXPLORATION 2. Consider the following numbers

$$11^0 = 1$$
$$11^1 = 11$$
$$11^2 = 121$$
$$11^3 = 1331$$
$$11^4 = 14641$$
$$11^5 = 161051$$
$$11^6 = 1771561$$

and compare them to the rows of Pascal's triangle. Precisely describe the pattern you see, and explain why it happens. What is the relationship between 101^n and Pascal's triangle? What about 1001^n?

EXPLORATION 3. Generate the table of *harmonic differences* by making $1/1, 1/2, 1/3, 1/4, \ldots$ the first row, and in each subsequent row record the differences of the adjacent numbers:

$$
\begin{array}{ccccccccccccc}
\frac{1}{1} & & \frac{1}{2} & & \frac{1}{3} & & \frac{1}{4} & & \frac{1}{5} & & \frac{1}{6} & & \cdots \\
& \frac{1}{2} & & \frac{1}{6} & & \frac{1}{12} & & \frac{1}{20} & & \frac{1}{30} & & \cdots & \\
& & \frac{1}{3} & & \frac{1}{12} & & \frac{1}{30} & & \frac{1}{60} & & \cdots & & \\
& & & \frac{1}{4} & & \frac{1}{20} & & \frac{1}{60} & & \cdots & & & \\
& & & & \frac{1}{5} & & \frac{1}{30} & & \cdots & & & & \\
& & & & & \frac{1}{6} & & \cdots . & & & & &
\end{array}
$$

Rotate the table so that $\frac{1}{1}$ is on top and the next row is $\frac{1}{2} \quad \frac{1}{2}$, then $\frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{3}$, *etc*. Then multiply the first row by 1, the second by 2, the third by 3, *etc*. What is the relationship between this new table and Pascal's triangle? Prove it.