PROBLEM 1. Consider the following relations on the set $\mathbb{R}$ of real numbers: inequality ($\neq$), strictly greater than ($>$), and less than or equal to ($\leq$). Determine what (if any) of the three properties of and equivalence relation — reflexive, symmetric, transitive — these relations have.

PROBLEM 2. Consider the relation $\sim$ on $\mathbb{R}$ such that $x \sim y$ if and only if $x - y$ is an integer. Prove that $\sim$ is an equivalence relation. What does a generic element of $\mathbb{R}/\sim$ look like?

Recall that for $\simeq$ an equivalence relation on set X, $X/\simeq$ is the set of equivalence classes for $\simeq$.

PROBLEM 3. Interpret and solve the following question using the language of equivalence classes:

QUESTION: A total of n Americans and n Russians attend a meeting and sit around a round table. If Americans and Russians alternate seats, in how many ways may they be seated up to rotation?

PROBLEM 4. We place two red and two black checkers on the corners of a square. Say that two configurations are equivalent if one can be rotated to the other. Check that this is an equivalence relation, and write down its equivalence classes. Can the number of equivalence classes be found by dividing 6 (the number of words with exactly two R's and two B's) by some natural number?

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

In the notation of Problem 2, does $\mathbb{R}/\sim$ have a natural “shape”?