PROBLEM 1. Suppose $k, n \in \mathbb{N}$ with $k \leq n$. Give two proofs that

$$\binom{n}{k} = \binom{n}{n-k}.$$

The first proof should be algebraic, using the defining formulas. The second should explain why both sides of the equality count the same thing.

PROBLEM 2. Let $a, b \in \mathbb{N}$. Prove that the number of NE lattice paths from $(0,0)$ to (a, b) is

$$\binom{a+b}{a} = \binom{a+b}{b}.$$

PROBLEM 3. Show that there are $1,098,240$ one-pair poker hands.

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

Ten ants are dropped in random positions on a meter-long stick. Some of these ants are initially traveling to the left and some are traveling to the right, but all travel at one meter/minute. When two ants meet, they bounce off of each other and change their directions (instantaneously). When an ant reaches the end of the stick, it walks off, never to return. What is the maximal amount of time (over all possible initial conditions) before the stick to be ant free? Characterize all initial conditions that achieve this maximal time. (If you have seen this problem before, do not spoil it for others in your group!)