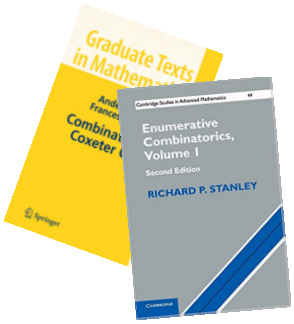
PROBLEM 1. You have nine math books. Five of them are yellow Springer-Verlag texts and four are gray Cambridge University Press texts.



(i) How many ways are there to arrange the books, left to right, along a shelf?
(ii) What if the yellow books need to stay together (but their ordering is still important)?
(iii) What if, in addition, the gray books need to stay together (and ordering within each color group is important)?

PROBLEM 2. A domino is a list of two, not necessarily distinct, numbers a, b where each of a and b are between 0 and 6, inclusive. We consider the pairs a, b and b, a to be the same.

(i) How many dominoes are there?

(ii) Say two dominoes *match* if they share at least one number. Thus, a matching pair will have the form

$$[a|b]\ [b|c]$$

where a, b, c are numbers between 0 and 6, inclusive. How many pairs of matching dominoes are there (where the order of the pair of dominoes does not count)? [Hints: A *double* is a domino with a repeated number, e.g., $[4|4]$. Why can't a matching pair consist of two doubles? Break the problem into two cases depending on whether a double occurs.]

PROBLEM 3.

(i) You are in an imaginary country in which coins come in denominations of $1, 2, \ldots, 7$ cents. In how many different ways can you pay for an item that costs 7 cents?

(ii) The next country you visit has only 5 and 11 cent coins. Thus, for instance, there is no way to create change for 13 cents. Consider all the (nonnegative integer) amounts that cannot be formed from collections of these coins. Is this set finite or infinite? If it is finite, what is its largest element?

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

(i) In Problem 1, what if the only restriction is that the colors appear in a symmetrical pattern about the central book? [Hint: Let g stand for gray and y for yellow. Suppose the first four books have the color pattern $ggyy$. What is the rest of the pattern? How many arrangements have this color pattern? How many possible color patterns are there for the first four books?]

(ii) In Problem 3, what if the denominations are a and b instead of 5 and 11? Can you come up with a formula for the largest amount that cannot be formed from these coins?

To read more about this fascinating problem, see the Wikipedia page on the coin problem. Spoiler alert: the article contains a solution to Challenge Problem (ii).