PROBLEM 1. Consider the following sets:

$$\begin{aligned}
A &= \{x \in \mathbb{Z} \mid x^2 \in \mathbb{N}\}, \\
B &= \{x \in \mathbb{N} \mid x \text{ is even}\} \cap \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\}, \\
C &= \{x \in \mathbb{N} \mid x \text{ is even}\} \cup \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\}, \\
D &= \{x \in \mathbb{N} \mid x \text{ is even}\} \triangle \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\}.
\end{aligned}$$

Write out some elements of each set and then describe the set in words, justifying your answer.

PROBLEM 2. Suppose that A and B are finite sets with $|A| = m$, $|B| = n$, and $m \leq n$. What are the smallest and largest possible values of $|A \cap B|$?

PROBLEM 3. Recall that De Morgan's law states that for all sets A, B, C,

$$C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$$

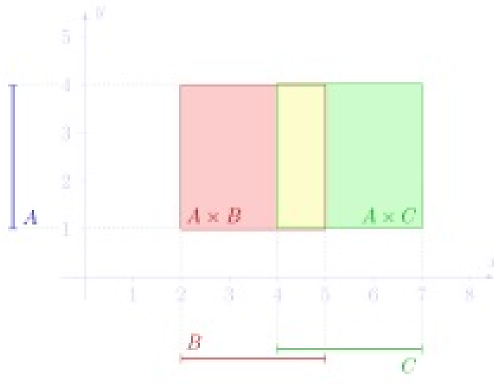
and

$$C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B).$$

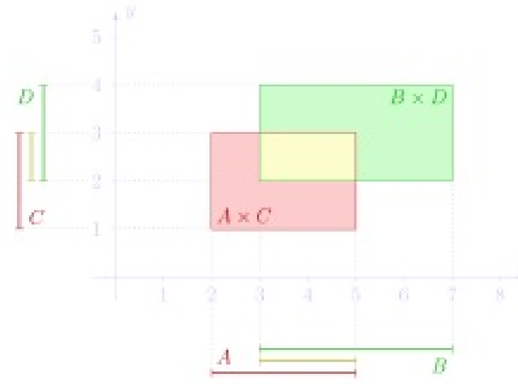
(i) Draw Venn diagrams that express these identities.
(ii) Prove the first identity.

In order to prove an equality of sets $X = Y$, you can show $X \subseteq Y$ and $Y \subseteq X$.

PROBLEM 4. Explain how the following pictures illustrate the indicated identities, and then prove one or both of them.



$A \times (B \cap C) = (A \times B) \cap (A \times C)$



$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$