MATH 113: DISCRETE STRUCTURES
FRIDAY WEEK 1 HANDOUT

Problem 1. Is it always the case that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$? Draw a picture to support your assertion and then prove it.

Cartesian product. There is another operation on sets called the *Cartesian product*. For sets A and B, their Cartesian product is the set

$$A \times B = \{(a, b) \mid a \in A, b \in B\},$$

the collection of ordered pairs where the first element is in A and the second is in B.

Question 2. Big Brothers Big Sisters of Portland has a collection A of 30 adult volunteers and group C of 50 children in need of an adult partner. What is a set which describes the possible adult-child pairings? How many adult-child pairings exist?

Problem 3. Find a general formula for $|A \times B|$ in terms of $|A|$ and $|B|$.

Functions. Functions are ways of relating one set to another. Thus to each element a of a set A, a function assigns exactly one element $b \in B$. If the function's name is f, then we write $b = f(a)$.

The set A is called the *domain* of f and B is its *codomain* (aka *range*). This can all be compactly expressed via the notation $f : A \to B$.

Each function $f : A \to B$ has an associated *graph* $G_f = \{(a, f(a)) \mid a \in A\} \subseteq A \times B$. A generic subset $G \subseteq A \times B$ is the graph of a function if and only if for each $a \in A$ there is a unique $b \in B$ such that $(a, b) \in G$. In set theory (which aims to express every mathematical concept in terms of sets), a function is actually defined to be such a special subset of $A \times B$. It's good to be aware of this formalism, but more useful in everyday mathematical practice to think of functions as assignments.

Problem 4. Which of the following subsets of $\{1, 2, 3\} \times \{a, b, c, d\}$ are functions?

(a) $\{(1, a), (2, b), (3, d)\}$
(b) $\{(2, d), (3, c)\}$
(c) $\{(1, b), (2, c), (3, a), (2, d)\}$
(d) $\{(1, a), (2, a), (3, a)\}$

Problem 5. Suppose $|A| = m$, $|B| = n$. How many functions $A \to B$ are there?