Lecture 7

Wednesday, February 4, 2015 7:59 AM

Quiz 1 — 12 minutes

<u>Theorem</u> $\binom{n}{k}$ is the number of possible k-member teams in a club with n members.

$\binom{45}{11}$ = # of different squads on a 45-member team you could put on football field for a play.

<u>Pf</u> (by induction on n): Base case n = 0 :

$\binom{0}{k} = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}$ and there is precisely one way to field a 0-member team from a club w/ 0 members.

For some n > 0 assume for induction that

$\binom{n-1}{k}$ = # of k-member teams from an (n-1)-sized club for any k.

Consider # of k-member teams from an n-sized club.

$\boxed{1 \quad 2 \quad 3 \quad \cdots \quad n-2 \quad n-1}$ n

$\binom{n-1}{k}$ teams exclude n $\binom{n-1}{k-1}$ teams include n

Thus $\binom{n-1}{k} + \binom{n-1}{k-1} \underset{\text{P's id}}{=} \binom{n}{k}$ k-member teams from n-players. $\square$

$$\binom{45}{11} = \frac{45!}{11!\,(45-11)!}$$

Note $\binom{n}{k}$ does not distinguish order.

Sets A *set* is collection of objects. The objects constituting a set are called its *members* or its *elements*.

If A is a set, write $m \in A$ when m is a member of A

$m \notin A$ when m is not a member of A.

e.g. (1) $A = [0, 1)$ is the collection of real #s x s.t. $0 \leq x < 1$

$\frac{1}{2} \in A \quad \frac{3}{\pi} \in A \quad 0 \in A \quad 1 \notin A \quad -15.2 \notin A$

~~(2) $A = [0,0] = \{0\}$~~ $\{1,2,3\}$

~~$\{x \mid x$ is a person in this room$\}$~~

(2) If we can list the elements of A, then enclosing them in "curly braces" $\{ \quad \}$:

- $\{0,1,2\}$
- $\{0,0,0,1,2,2\} = \{0,1,2\}$
- $\{$ cat, dog, mouse, $15\}$
- $\{\{0,1,2\}\} = A$:

$$\{0,1,2\} \in A$$
$$0 \notin A$$
$$0 \in \{\{0,1,2\}, 0\}$$

(3) $\{\}$ is denoted $\emptyset$.

For any x, $\quad x \notin \emptyset$

$$\emptyset \notin \emptyset$$

$\{\emptyset\} \neq \emptyset \qquad \emptyset \in \{\emptyset\}$

$$\emptyset \notin \emptyset$$

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⑤ Propositional def'ns.

- $\{x \mid P(x)\}$ is the collection of x s.t. $P(x) = T$

$\{x \in A \mid P(x)\}$ — " — $x \in A$ — " —

e.g. $\{x \in \mathbb{R} \mid 0 \leq x < 1\} = [0, 1)$