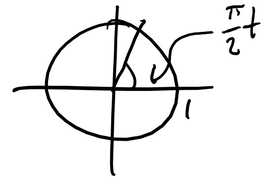
Lecture 48

Wednesday, April 29, 2015 8:05 AM



Thus for $t \in \mathbb{R}$,

$$C(st) + iS(st) = E(ist) = \cos\left(\frac{\pi}{2}t\right) + i\sin\left(\frac{\pi}{2}t\right)$$

$$\Longrightarrow \quad C(st) = \cos\left(\frac{\pi}{2}t\right)$$

$$S(st) = \sin\left(\frac{\pi}{2}t\right)$$

$$\Longrightarrow \quad C\left(\frac{2s}{\pi}t\right) = \cos(t)$$

$$S\left(\frac{2s}{\pi}t\right) = \sin(t)$$

$$\Longrightarrow \quad \cos'(t) = C'\left(\frac{2s}{\pi}t\right)\cdot\frac{2s}{\pi} = -\frac{2s}{\pi}S\left(\frac{2s}{\pi}t\right)$$

$$= -\frac{2s}{\pi}\sin(t)$$

$$\sin'(t) = \frac{2s}{\pi}\cos(t)$$

$$\frac{2s}{\pi} = \frac{2s}{\pi}\cos(0) = \sin'(0) = \lim_{x\to 0^+}\frac{\sin x - \sin 0}{x}$$

$$= \lim_{x\to 0^+}\underbrace{\frac{\sin x}{x}}\ \underset{\text{squeeze}}{=}\ 1 \Longrightarrow \boxed{s = \frac{\pi}{2}} \qquad \square$$

For $x > 0$, close enough to 0, $\quad \cos x < \frac{\sin x}{x} < \frac{1}{\cos x}$, $\quad x \to 0^+$

$\cos x \to 1$, $\quad \frac{1}{\cos x} \to 1$

Thm If $z(r,\theta)$ is the cpx # w/ length r & angle θ from pos real axis, then

$$z(r,\theta) = r \cdot E(i\theta). \qquad \square$$

Notation For $z \in \mathbb{C}$, we write e^z for $E(z)$

i.e.

$$\begin{array}{ccc} e^{a+bi} & = & e^a \cdot \underbrace{e^{ib}} \\ \| & & \\ E(a+bi) & & \\ \| & & \\ E(a)\cdot E(ib) & & \end{array}$$

cpx # of length 1 angle b from pos real axis.

Call e^z the <u>complex exponential</u>.