Lecture 33

Sunday, March 29, 2015 11:27 AM

Convergence of sequences

Defn $s = (s_n)$ <u>converges</u> to $L \in \mathbb{C}$ if $\forall \varepsilon > 0$ $\exists N$ s.t. $\forall n \in \mathbb{Z}^+$, $n > N$, $|s_n - L| < \varepsilon$.

Call L the <u>limit</u> of (s_n), write

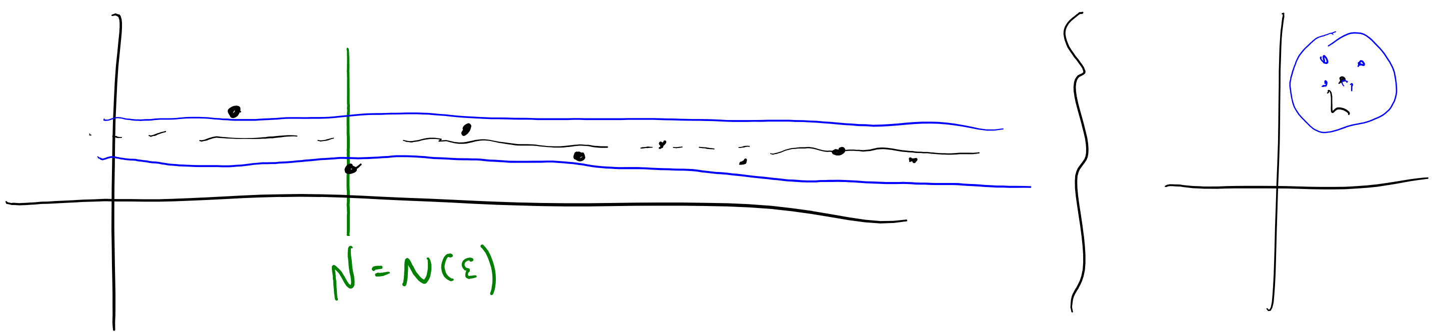
$$s_n \longrightarrow L, \quad (s_n) \to L$$

$$\lim s = L, \quad \lim_{n \to \infty} s_n = L$$

e.g. $(s_n) = (c) \longrightarrow c$

Pf Given $\varepsilon > 0$ let $N = 1$. Then for $n > N$,

$|s_n - c| = |c - c| = 0 < \varepsilon$, so $(c) \longrightarrow c$. $\square$



e.g. $\lim_{n \to \infty} \frac{1}{n} = 0$.

Pf Given $\varepsilon > 0$ set $N = \frac{1}{\varepsilon}$, a pos real number. If $n > N$, then $|s_n - 0| = |\frac{1}{n} - 0| = \frac{1}{n} < \frac{1}{N}$ (b/c $n > N > 0$)

$$= \frac{1}{1/\varepsilon} = \varepsilon. \quad \text{Thus } \frac{1}{n} \longrightarrow 0.$$

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$$\lim_{n\to\infty} \frac{2n+3n^2}{3+4n+n^2} = 3.$$

Scratch Let $\varepsilon > 0$. Set $N =$ ________, a pos real #. If $n > N$, then

$$\left|\frac{2n+3n^2}{3+4n+n^2} - 3\right| = \left|\frac{2n+3n^2 - 9 - 12n - 3n^2}{3+4n+n^2}\right|$$

$$= \left|\frac{-9-10n}{3+4n+n^2}\right|$$

$$= \frac{9+10n}{3+4n+n^2} \quad (\text{b/c } n > 0)$$

$$\leq \frac{9+10n}{n^2} \quad (\text{b/c } 3+4n+n^2 > n^2)$$

$$\leq \frac{n+10n}{n^2} \quad [\text{as long as } N \geq 8 \Rightarrow n \geq 9]$$

$$= \frac{11n}{n^2}$$

$$= \frac{11}{n} < \frac{11}{N}$$

$$\leq \varepsilon \quad [\text{as long as } N \geq 11/\varepsilon]$$

So $N = \max\{8, 11/\varepsilon\}$ will work.

... $\square$