REVIEW

Friday, March 6, 2015 8:06 AM

<u>Claim</u> For p a prime number, $\mathbb{Z}/p\mathbb{Z}$ is a field, but not an ordered field.

<u>Pf</u> We'll assume $\mathbb{Z}/p\mathbb{Z}$ is a field. Why not ordered?

Need to show there is <u>no</u> rel'n $<$ on $\mathbb{Z}/p\mathbb{Z}$

s.t.

1. $<$ transitive
2. trichotomy: $\forall x, y \in \mathbb{Z}/p\mathbb{Z}$, $\underbrace{\,}_{\text{exactly one of}}$ $x<y$, $x=y$, or $x>y$ is true
3. $<$ respects $+$
4. $<$ respects $\cdot$ by positive elts.

Proceed by $\cancel{\circ}$: assume such a rel'n exists.

We have shown that $0<1$ in any ordered field, so $[0]<[1]$. Adding $-[1]$ to both sides, we get

$$[0]+(-[1]) < [1]+(-[1])$$

$$-[1] < [0]$$

some way to get $[p-1] < [0]$ ~~, then add $[1]$ to get $[0] < [1]$?~~ and also know $[0] < [p-1]$

Note that $[p-1]+[1]=[p]=[0] \in \mathbb{Z}/p\mathbb{Z}$

so $[p-1] = -[1]$ and thus

$$[p-1] < [0].$$

Adding $[1]$ over and over to $[0] < [1]$, we get

$$[1] < [2]$$
$$[2] < [3]$$
$$[3] < [4]$$
$$\vdots$$
$$[p-2] < [p-1]$$

By transitivity,

$$[0] < [p-1].$$

This contradicts trichotomy so there is no order on $\mathbb{Z}/p\mathbb{Z}$. $\square$

Pascal's $\triangle$

$$
\begin{array}{ccccccccccc}
 & & & & 1 & & & & & & 0\\
 & & & 1 & & 1 & & & & & 1\\
 & & 1 & & 2 & & 1 & & & & 2\\
 & 1 & & 3 & & 3 & & 1 & & & \\
1 & & 4 & & 6 & & 4 & & 1 & &
\end{array}
$$

$\binom{4}{2}$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \qquad \binom{4}{2} = \frac{4!}{2!\,2!}$$

$$= \frac{4\cdot 3\cdot \cancel{2\cdot 1}}{\cancel{2\cdot 1}\cdot 2\cdot 1}$$

$$= 2\cdot 3 = 6$$

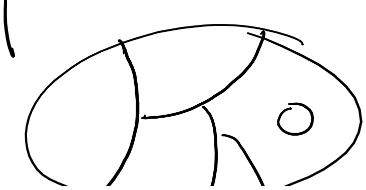
Relations

A <u>relation</u> on A and B is a subset of $A\times B$

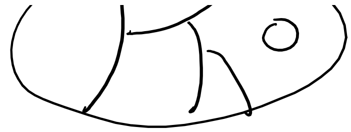
A <u>relation</u> on A is a subset of $A\times A$.

<u>Equiv relns</u>: Relations satisfying reflexivity, symmetry, and transitivity.

Equiv classes of an equiv reln on A partition A

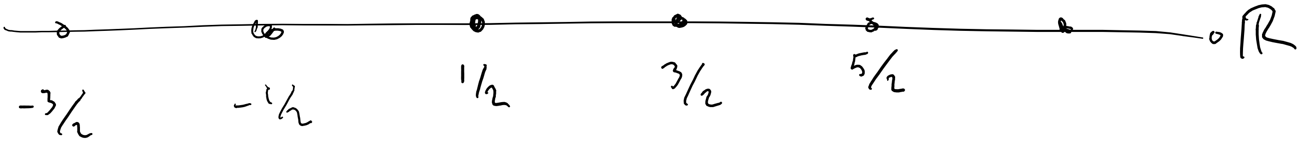


A



Equiv rel'n on $\mathbb{R}$ defined by $x \sim y$ iff $x - y \in \mathbb{Z}$

Prob Show that $\forall x \in \mathbb{R}$, $\exists b \in [0,1)$ s.t. $x \sim b$.



$x \sim 0$? $\iff$ $x - 0 \in \mathbb{Z}$ so no.

$[1/2] = \{ \frac{1}{2} + n \mid n \in \mathbb{Z} \}$

$[x] = \{ x + n \mid n \in \mathbb{Z} \}$ b/c $x \sim y \iff x - y = n \in \mathbb{Z}$

$\iff y = x - n$

Problem is equiv to $[x] \cap [0,1) \neq \emptyset$.

$[2.45] \cap [0,1) = \{0.45\}$

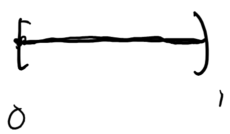
$\{2.45, 3.45, 4.45, 5.45, \ldots, 1.45, 0.45, -0.55, \ldots\}$

$[783.1111112] \cap [0,1) = \{0.1111112\}$

$[-0.3] \cap [0,1) = \{0.7\}$

$$\mathbb{R}/\sim \; = \{ [x] \mid x \in \mathbb{R} \}$$

$$= \{ [x] \mid x \in [0,1) \}$$



— a circle!

F a field, $x, y \in F \implies (-x)y = -(xy)$.

Pf Know $x + (-x) = 0$ by definition.

In order to show $(-x)y = -(xy)$, we must show $xy + ((-x)y) = 0$.

Observe, $xy + ((-x)y) = (x + (-x))y$

$$= 0 \cdot y$$

$$= 0,$$

as desired, so $(-x)y = -(xy)$. $\square$

Q How do we show $y = x^{-1}$?

A Prove that $xy = 1$.

$(-x)^{-1} = -(x^{-1})$