MATH 112: EXAM 1

Name: ______________________________

Instructions. Take-home exam. 60 minutes. Closed everything except blank scratch paper and one two-sided 8.5″×11″ page of notes.

Do not read beyond this page until you are seated in a distraction-free environment. Once you read beyond this page, your time has started and you have 60 minutes to complete the exam. All of the work must be your own and you may not rely on any outside resources. *The Honor Principle prevails.*

Please write your name in the space provided above. Please provide solutions (as cleanly presented as possible) on the front side of each page. The back side of each sheet of paper is intended for scratch work and **will not be graded** unless there is an explicit note directing the instructor to do so.

Once your 60 minutes are up, scan your work and email it to `ormsbyk@reed.edu` with subject line Math 112 Exam 1. (Please first send the scan to yourself, check that the pdf appropriately reflects your work (*i.e.*, is legible and contains all pages), rename the file along the lines of `MyNameExam1.pdf`, and then follow the above email instructions.) **Save the paper copy of your exam** until you have received your graded copy of Exam 1!

*Your scan is due by 11:59*PM *on Monday, March 9. Remember that we just started daylight savings time!*

Problem	Points	Out of
1		28
2		20
3		20
4		15
5		17
Total		100

Date: 9.III.15.

Problem 1 (28 points). For the following statements, circle T if the statement is true and F if the statement is false. You do not need to show any work.

$(P \implies Q) \implies (Q \implies P)$	T or F
Every element of a field has a multiplicative inverse.	T or F
$((P \implies Q) \wedge P) \implies Q$	T or F
Sometimes it's OK to divide by 0.	T or F
There exists an inductive set which is a proper subset of $\mathbb{N}$.	T or F
If the set A has n elements, then there are 2^{n^2} relations on A.	T or F
If the set A has n elements, then there are 2^{n^2} equivalence relations on A.	T or F
In $\mathbb{C}$, $(a+bi)+(c+di)=(a+c)+(b+d)i$.	T or F
In $\mathbb{C}$, $(a+bi)\cdot(c+di)=(a\cdot c)+(b\cdot d)i$.	T or F
If $g : D \to C$ is a function, then the domain of g is D.	T or F
$\neg(\forall x, \exists y \text{ such that } P(x,y)) \iff (\exists x \text{ such that } \forall y, \neg P(x,y))$	T or F
Every subset of the real numbers has a supremum.	T or F

For the following two statements, let F denote the set of functions from $\{1,2,3,4\}$ to $\{1,2,3,4\}$.

Composition is an associative binary operation on F	T or F.
Composition is a commutative binary operation on F	T or F.

Problem 2 (20 points). Let $f : A \to B$ be a function with domain A and codomain B. Define a relation $\simeq_f$ on A so that for $x, y \in A$, $x \simeq_f y$ if and only if $f(x) = f(y)$.

(a) Prove that $\simeq_f$ is an equivalence relation.

(b) Consider the function $n : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ defined by $n(z) = z/|z|$. Describe the equivalence classes of $\simeq_n$ in words *and* in a picture.

Problem 3 (20 points). (a) Use mathematical induction to show that

$$\sum_{i=1}^{n}(2i-1) = n^2$$

for any integer $n \geq 0$.

(b) Given part (a) and that $\sum_{i=1}^{m} i = \frac{m(m+1)}{2}$ for all integers $m \geq 0$, find a closed formula for

$$\sum_{i=1}^{n} 2i.$$

Problem 4 (15 points). Use the axioms for a field to prove that in any field F, if $x \in F \setminus \{0\}$, then $(-x)^{-1} = -(x^{-1})$.

Problem 5 (17 points). Recall that the real numbers $\mathbb{R}$ are the unique complete ordered field. Let $S = \{x \in \mathbb{R} \mid x^2 < 2\}$, and let $T = \{5 - 1/2^n \mid n \in \mathbb{N}\}$.

(a) Find $\sup S$, $\sup T$, and $\sup(S \cup T)$. (No proofs necessary.)

(b) Now suppose A and B are arbitrary bounded above subsets of $\mathbb{R}$. Find a formula for $\sup(A \cup B)$ in terms of $\sup A$ and $\sup B$; prove that your formula is correct.