MATH 111: A TRIG LIMIT VIA THE SQUEEZE THEOREM

Our goal is to prove that

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

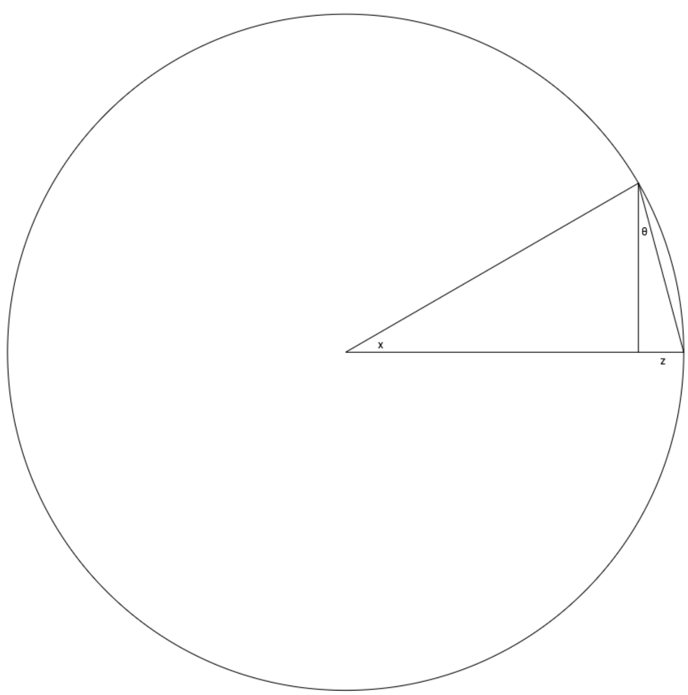
via the squeeze theorem. Below I sketch some steps that will produce functions $h(x)$ and $g(x)$ such that $h(x) \leq \frac{1-\cos x}{x} \leq g(x)$ (at least when $0 < x < \pi/2$) and $\lim_{x\to 0} h(x) = 0 = \lim_{x\to 0} g(x)$. Feel free to follow these suggestions in you proof, or come up with something new!

Warning: In your proof, you will also have to carefully consider what happens when $-\pi/2 < x < 0$. It might be easier to prove that

$$\lim_{x\to 0} \left|\frac{1-\cos x}{x}\right| = 0$$

and then invoke the result you proved in Exercise 112(a) from §1.3 in you previous problem set.

Consider the picture below in which x is an angle (measured in radians) between 0 and $\pi/2$ and the circle is a unit circle.



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Step 1: Find a basic arithmetic expression involving trig functions for the length of side z. (When z is mentioned below, you'll want to plug in your new expression for it.)
Step 2: Recall the relationship between radians and arclength (on the unit circle) in order to conclude that $\sin\theta \geq \frac{z}{x}$.
Step 3: Prove that the angle labeled θ is equal to $\frac{x}{2}$, so that now $\sin\frac{x}{2} \geq \frac{z}{x}$.
Step 4: Prove that $\frac{z}{x} \geq 0$ as well.