Index

absolute convergence, 221	axiom, completeness, 97
absolute summability, 221	axioms, 2
absolute value, 48	for a field, 29
of complex number, 106	for ordered field, 43
product formula for, 49	axis,
quotient formula for, 51	imaginary, 109
addition,	real, 109
addition (field operation), 29	1001, 100
addition in \mathbf{Z}_n , 27	Bernoulli, Jacob (1654–1705), 32, 224,
addition laws for sine and co-	259
sine, 243	between, 179
addition of inequalities, 45	Bhaskara (born 1114–1185), 63
Alembert, Jean (1717–1783), 224	binary
alternating series, 216	operation, 19
alternating series test, 216	search sequence, 94
ambiguous,	Bolzano, Bernhard, (1781–1848), 181
a^{b^c} , 227	Boole, George (1815–1864), 29
sequence notation, 92	bound,
and, logical connective, 8	for a function, 174
Archimedean property, 97, 98, 105	for a set, 173
Archimedes, 105	for sequence, 139
argument (of a complex number), 247	lower, 154
Aristotle (384-322 B.C.), 31, 69	upper, 154
Arnold, Vladimir (1937–??), 124	bounded,
cats, 124	function, 173
Ars Magna, 90	sequence, 139, 143
Artin, Emil (1898–1962) 54	set, 173
associative operation, 21, 29	boundedness theorem, 178
associativity of \odot , 84, 267	bug, 26
associativity of \oplus , 267	Buhler, Joe (1950–??), 4
J - ,	

calculator operations, 26 cancellation law, 23, 33 Cardano, Girolamo (1501–1576), 54, 90 Cartesian product, 16 cat, Arnold, 124 discontinuous image of, 172 inverse of, 121 square of, 120 exponential of, 249 Cauchy, Augustin (1789–1857), 82, 91, 181 chain rule, 185 circle. 110 of convergence, 232 unit, 111 Clenias, 68 closed interval, 93 codomain of function, 16 commensurable, 69 commutative operation, 24, 29 commutativity of addition in field, 35 comparison test for series, 206 limit comparison test, 211 comparison theorem for null sequences, 131 completeness, 97, 105 completeness axiom, 97	conjugate, complex, 89, 91 constant sequence, 127, 128 continuity 105 of exp, (in theorem 12.27), 238 of roots, 164 of ln, 240 continuous, 161 at a point, 161 on a set, 161 convergence, 129 of search sequence, 94 absolute, 221 circle of, 232 disc of, 232 of geometric sequence, 145 of geometric series, 145 radius of, 232 convergent sequence, 127, 143 limit of, 138 product theorem, 137 quotient theorem, 141 reciprocal theorem, 141 reciprocal theorem, 137 uniqueness theorem, 138 copy (of F in C _F) 87 cosine, 235, 242, 250 complex, 222 critical point, 187 theorem, 188
complex, conjugate, 89 function, 161 numbers, 106 complexification, 83 composition, 159 of continuous functions, 166 of functions, 165	D'Alembert, Jean (1717–1783), 224 De Moivre, Abraham (1667–1754), 124 De Moivre's formula, 113 decimal notation, 146 decimals, 1, decomposition theorem, 136

decreasing,	entertainments, 3
function, 101	Epicureans, 110
sequence, 153	equality,
definition by recursion, 71	of functions, 16
derivative, 182	of objects, 12
Descartes, Rene, 1596–1650, 82	of ordered pairs, 15
Dickson, Leonard Eugene, (1874–1954),	of rules, 16
31	of sets, 7
${ m difference},$	reflexive property of, 12
of sets, 15	symmetric property of, 12
symmetric, 21	transitive property of, 12
differential equation (12.23), 236, 237	substition property for, 12
differentiation of power series, 234,	${\it equivalence},$
253	of sets of propositions, 43
digits, 37	of propositions, 11
direction in \mathbf{C} , 114, 115	Euclid (365–300BC??), 64, 69, 105,
$\operatorname{disc},$	110
of convergence, 232	Euler, Leonard (1707–1783), 91, 124,
closed, 111	208, 260
open, 110	summation notation, 82
unit, 111	$\operatorname{even},$
distance (in ordered field), 51	integer, 66
distributive law 29, 30	number, 63
in a field, 39	exercises, 3
in \mathbf{Z}_n , 268	exponential function, 238
divergence,	exponents, rational, 104, 105
of search sequence, 94	extreme value theorem, 177
of sequence, 127	factorial function, 71, 82
test for, 152	factorization, 79
division in a field, 39	of $a^p - r^p$, 78
domain of function, 16	feature, 26
double inverse theorem, 22, 34	field,
draughts, 68	axioms for, 29
dull sequence, 129	orderable, 43
e, 238, 260	real, 97
empty set, 6	rear, 91
empty set, o endpoints of an interval, 52	
enapoints of an interval, 02	

functions, 16, 70 addition and multiplication of, 100 complex, 161 increasing and decreasing, 101 Gauss, Carl (1777-1855), 253 geometric sequence, 125, 203 convergence of, 145 geometric series, 77, 126, 203 convergence of, 145 graph, 100 Gregory, John (1638–1675), 225, 258 Göttingen, 224 Hamilton, William R. (1805–1865), 29, 91 harmonic series, 203, 224 Huntington, Edward (1874–1952), 43, 54 hyperbolic functions, 244 identity element for binary operation, 20 image of a function, 119 imaginary axis, 109 imaginary part of complex number, 107 implication, 9 incommensurables, 68 increasing, function, 101	infinite series, 202 integers, 70 even, 66 odd, 66 in a field, 64 informal definition, 6 interior, 190 interior point, 190, intermediate value theorem, 179, 180 intersection, 15 interval, 52 closed or open, 93 inverse, for binary operation, 21 of cat, 121 invertible element for binary operation, 22 Jones, William, (1675–1749), 260 Koch, Helge von (1870–1924), 158 Kramp, Christian, (1760–1826), 82 Lagrange, Joseph, (1736–1813), 224 Landau, Edmund (1877–1938), 224 laws, of exponents (fractional), 104 of signs, 46 Laws of Thought, 29 The Laws, 68 least element principle, 60 Leibniz, Gottfried (1646–1716), 224,
function, 101	Leibniz, Gottfried (1646–1716), 224,
sequence, 153 induction theorem, 57 generalized, 62 inductive, 55 inequalities, addition of, 45, 47 inequality theorem, 149 inequality, triangle, 50	225, 258 length of complex number, 114

limit,	Newton, Isaac (1643–1727), 105, 201,
comparison test, 211	225
of a function, 167	not, negation, 8
of a sequence, 138	null sequences, 129
point, 166	comparision theorem for, 131
uniqueness of, 168	root theorem for, 132
line segment, 192	sum theorem for, 134
logarithm, 240	product theorem for, 136
lower bound,	null-times-bounded theorem, 139
for a sequence, 154	number,
for a set, 178	complex, 106
,	even, 63
Madhava of Sangamagramma (c. 1340-	natural, 56, 70
$1425),\ 201,\ 225,\ 258$	odd, 63
Mahavira (ninth century), 63	rational, 70
Maple, 25, 82, 91, 245, 260	real, 97
maps to, 125	,
Mathematica, 82, 91, 245, 260	odd,
maximizing set, 176	integer, 66
max function, 80, 81	number, 63
maximum, 176	open interval, 93
critical point theorem, 188	opposite sign, 46
mean value theorem, 189	or, logical connective, 8
Mercator, Nicolaus (1620-1687), 225	orderable field, 43
midpoint, 94	ordered field,
minimum, 176	axioms for, 43
monotonic sequence, 153	completemess of, 97
multiplication	ordered,
(field operation), 29	pair, 15
of inequalities, 47	$ ext{triple}, 15$
table, 27	Oresme, Nicole (1323–1382), 224
in \mathbf{Z}_n , 27	. 1 145
Mycielski, Jan, 157	pair, ordered, 15
	paradox, 17
natural numbers, 56, 70	parentheses, 31
informal definition, 6	Pascal, Blaise (1623-1662), 64
negative elements in ordered field,	path, 192
44	Peano, Giuseppe (1858–1932), 64

periodicity, of sin and cos, 12.59 h) and i), 246 of exp, 245 Philitas of Cos, 18 pi (π), 244, 260 Plato (427?–347B.C.), The Laws, 68 polar decomposition, 114 polygon representation for a complex sequence, 125 polygon, snowflake, 148 positive elements in ordered field, 43	quotient, formula for absolute value, 51 of functions, 100 rule for differentiation, 187 theorem, for continuous functions, 163 for convergent sequences, 141 radius of convergence, 232 ratio, for a geometric sequence, 125 for geometric series, 126 test, 212, 224
power, function, 72, 82 integer, 74 rule for differentiation, 187 series, 226 precedence, 31 precision function, 130, 157 Priora Analytica, 69 Proclus, 110 product, formula for absolute value, 49 of functions, 100 rule for differentiation, 187 theorem, for continuous functions, 163 for convergent sequences, 137 for limits of functions, 172 for null sequences, 136 propositions, 7 equivalence of, 11	rational, exponents, 105 numbers, 70 in a field, 65 informally defined, 6 in sense of Euclid, 69 real axis, 109 real field, 97 real part of comlex number, 107 reciprocal rule for differentiation, 186 reciprocal theorem for convergent sequences, 140 recursion, 71 Reed College, 4 reflexivity of equality, 12 restriction theorem, 192 reverse triangle inequality, 139 Rolle, Michel (1672–1719), 201 Rolle's theorem, 189 root,
proposition form, 14 Pythagorean theorem, 109 quadratic formula, 42	of complex number, 113, 249 of real number (theorem 5.49), 103 theorem for null sequences, 132
- ,	continuity of roots, 164

Sangamagramma, Madhava (c. 1340-1425), 201 Schreier, Otto (1901-1929), 54 search sequence, 94 convergence of, 94 segment, 192 sequence, 92, 125 bounded, 139, 143 constant, 127, 128 convergent, 127, 129, 143 decreasing, 153 divergent, 127 dull, 129 increasing. 153 lower bound for, 154 null, 129 search, 94 summable, 203 upper bound for, 154 series, 202 alternating 216 operator, 202 power series, 226 sum of, 203 Servois, François-Joseph (1767-1847), 29 set, 5 empty set, 6 set difference, 15 sine 235, 242, 250 complex, 222 Skolem functions, 157 snowflake, 148, 158 subset, 7 substition property of equality, 12 of functions, 100 of a series, 203 theorem, for continuous functions, 163 for convergent sequences, 137 for differentiable functions, 183 for series, 204 summable, sequence, 203 absolutely, 221 summation, 76 function, 82 symmetric difference, 21 symmetry of equality, 12 of implication, 9 translate of a sequence, 149 translation theorem, 149 translation for series, 203 absolutely, 221 summable, sequence, 203 absolutely, 221 summation, 76 functions, 82 symmetric difference, 21 symmetry of equality, 12 of implication, 9 translate of a sequence, 149 translation theorem, 180 for convergent sequences, 137 for differentiable functions, 183 for convergent sequences, 137 for differentiable functions, 183 for series, 203 theorem, for continuous functions, 163 for convergent sequences, 137 for differentiable functions, 183 for series, 203 theorem, for continuous functions, 163 for convergent sequences, 137 for differentiable functions, 183 for series, 203 theorem, for contin
250

```
triple, ordered, 15
union, 15
uniqueness,
   of R, 97
   of identities, 20
   of inverses, 22
    of limits, 168
    theorem for convergent sequences,
        138
unit circle, 111
unit disc, 111
upper bound,
    for a sequence, 154
   for a set, 178
Waring, Edward, (1734–1798), 224
Weber, Heinrich Martin (1842–1913),
        43
Weierstrass, Karl (1815–1897), 54,
        181
zero-one law, 30
```