

# Appendix C

## List of Symbols

$\mathbf{N}$	natural numbers, 6, 69
$\mathbf{Z}$	integers, 6, 69
$\mathbf{Q}$	rational numbers, 6, 69
$\emptyset$	empty set, 6
$\mathbf{Q}^+$	positive rationals, 6
$a \in A$	$a$ is in $A$ , 6
$a \notin A$	$a$ is not in $A$ , 6
$A \subset B$	subset, 7
$\subset$	subset, 7
$A = B$	set equality, 7
$a \neq b$	$a$ is not equal to $b$ ., 9
$P \implies Q$	$P$ implies $Q$ , 9
$P \implies Q \implies R \implies S$	11
$P \iff Q$	11
$x = y$	12
$a = b = c = d$	13
$P(x)$ , proposition form	14
$\{x \in A : P(x)\}$	the set of all $x$ in $A$ such that $P(x)$ is true, 14
$R \cap T$	intersection of sets, 15
$R \cup T$	union of sets, 15
$R \setminus T$	set difference, 15

$(a, b)$	ordered pair, 15
$(a, b, c)$	ordered triple, 15
$A \times B$	Cartesian product, 16
$f : A \rightarrow B$	function with domain $A$ , codomain $B$ , 16
$\Delta$	symmetric difference, 21
$x^{-1}$	inverse for $x$ , 22
$C, \tilde{C}$	calculator numbers, 26
$\oplus$	calculator addition, 26
$\ominus$	calculator subtraction, 26
$\odot$	calculator multiplication, 26
$\oslash$	calculator division, 26
$\mathbf{Z}_n$	$\{x \in \mathbf{N} : x < n\}$ , 27
$\oplus_n$	addition in $\mathbf{Z}_n$ , 27
$\odot_n$	multiplication in $\mathbf{Z}_n$ , 27
$+$	addition in a field, 29
$\cdot$	multiplication in a field, 29
$-x$	additive inverse in a field, 30
$x^{-1}$	multiplicative inverse in field, 30
$\mathbf{Z}_n$	a finite field, 33
$\mathbf{D}_F$	set of digits in $F$ , 37
$x^2$	$x \cdot x$ , 38
$a - b$	$a + (-b)$ , 39
$a/b$	$a \cdot b^{-1}$ , 39
$\frac{a}{b}$	$a \cdot b^{-1}$ , 39
$F^+$	positive elements in ordered field, 43
$F^-$	negative elements in an ordered field, 44
$<, \leq, >, \geq$	order relations in an ordered field, 45
$ x $	absolute value, 48
$ x - y $	distance from $x$ to $y$ , 51
$\mathbf{N}_F$	natural numbers in $F$ , 56
$\mathbf{Z}_F$	integers in $F$ , 64
$\mathbf{Q}_F$	rational numbers in $F$ , 65
$n!$	factorial function, 71
$a^n$	power function, 72, 74
$\mathbf{Z}_{\geq k}$	$\{n \in \mathbf{Z} : n \geq k\}$ , 75, 93

$S(p) = \sum_{j=k}^p f(j)$	summation notation, 76
$\dots$	hidden induction, 77
$\max(p, q)$	maximum of $p$ and $q$ , 80
$\max_{j \leq n \leq l} f(n)$	maximum, 81
$\mathbf{Z}_{j \leq n \leq l}$	$\{n \in \mathbf{Z}: j \leq n \leq l\}$ , 81
$C_F$	complexification of $F$ , 83
$\oplus, \odot$	operations on $C_F$ , 84
$i$	square root of $-1$ , 87
$\tilde{a}, (a, 0)$	element of $\mathbf{C}_F$ , 87
$z^*$	complex conjugate of $z$ , 89
$\{f(n)\}$	sequence, 92
$\{f(0), f(1), f(2), \dots\}$	sequence, 92
$\{[a_n, b_n]\} \rightarrow x$	convergence of search sequence, 94
$\mathbf{R}$	real field, 97
$a^{\frac{1}{p}}$	$p$ th root of $a$ , 104
$\sqrt{a}$	square root of $a$ , 104
$a^r$	fractional power, 104
$\mathbf{C}$	complex numbers, 106
$ z $	absolute value, 106
$\operatorname{Re}(z)$	real part of $z$ , 107
$\operatorname{Im}(z)$	imaginary part of $z$ , 107
$C(\alpha, r)$	circle in $\mathbf{C}$ , 110
$D(\alpha, r)$	open disc, 110
$\bar{D}(\alpha, r)$	closed disc, 111
$n \mapsto 2^n$	maps to, 125
$\tilde{a}$	constant sequence, 127
$f \rightarrow L$	$f$ converges to $L$ , 127
$N_f$	precision function for $f$ , 130
$\operatorname{Re} f, \operatorname{Im} f, f^*,  f $	sequences, 134
$\lim f, \lim\{f(n)\}$	limit of a sequence, 138
$.a_1 a_2 \dots a_n$	decimal notation, 146
$g \circ a$	composition, 159

$\text{abs}(z)$	$ z $ , 162
$\text{conj}(z)$	$z^*$ , 162
$g \circ f$	composition, 165
$\lim_a f$	limit of $f$ at $a$ , 167
$\lim_{z \rightarrow a} f(z)$	limit of $f$ at $a$ , 167
$f'(a)$	derivative of $f$ at $a$ , 182
$D_a f$	182
$\text{int}(J)$	interior of $J$ , 190
$f _T$	restriction of $f$ to $T$ , 192
$\lambda_{ab}$	path, 192
$\Lambda_{ab}$	line segment, 192
$C_n$	cosine polynomial, 197
$S_n$	sine polynomial, 197
$\cos$	cosine, 198, 222
$\sin$	sine, 198, 222
$\sum f$	series corresponding to $f$ , 202
$\sum_{n=0}^{\infty} a_n$	sum of a series, 203
$\{H_n\}$	harmonic series, 204
$a^{b^c}$	227
$\exp(z)$	exponential function, 238
$e$	$\exp(1)$ , 238
$e^z$	exponential function, 239
$\ln(t)$	logarithm of $t$ , 240
$\sinh$	hyperbolic sine, 244
$\cosh$	hyperbolic cosine, 244
$\pi$	pi, 244
$\text{Arg}(z)$	argument of $z$ , 247