Appendix A

Hints and Answers

Exercise 3: The Rhind value is $256/81 = 3.1604\ldots$

Exercise 1.7: Look at the boundary.

Exercise 1.10: If a set has no endpoints, then it contains all of its endpoints and none of its endpoints.

Exercise 2.10: $1^3 + 2^3 + \cdots n^3 = \frac{n^2(n+1)^2}{4}$.

Exercise 2.18: $\text{area}T(a) = \dfrac{2}{3}a^{\frac{3}{2}}$.

Exercise 2.27: $.027027027\ldots = \frac{1}{37}$.

Exercise 2.36: I let $O_j = B(a^{\frac{j}{N}}, a^{\frac{j-1}{N}}; 0, a^{-\frac{2j}{N}})$ and $I_j = B(a^{\frac{j}{N}}, a^{\frac{j-1}{N}}; 0, a^{-\frac{2(j-1)}{N}})$.

Exercise 3.20: Recall $(R \Longrightarrow S) \Longleftrightarrow (S$ or not $R)$.

Exercise 5.61: $S_1^{ab}[\frac{1}{t}] = S_1^a[\frac{1}{t}] \cup S_a^{ab}[\frac{1}{t}]$. (Draw a picture.)

Exercise 5.80: Consider a partition with a fairly large number of points.

Exercise 6.33: The assertion is false.

Exercise 6.59: (part e) The limit is $\frac{1}{3}$. It simplifies matters if you factor both the numerator and the denominator. The sequence in part g) is a translate of the sequence in part f).

Exercise 6.69: All four statements are false.

Exercise 6.94: a) $(1+\frac{3}{n})^{2n} = ((1+3/n)^n)^2$.

Exercise 6.97: $(1-\frac{c}{n})^n = \frac{(1-\frac{c^2}{n^2})^n}{(1+\frac{c}{n})^n}$.

Exercise 7.16: Take $c = \frac{b}{a}$ in lemma 7.13.

Exercise 7.18: $A_0^a f = A_0^{\frac{1}{n}} f + A_{\frac{1}{n}}^a f$. Show that $A_0^{\frac{1}{n}} f$ is small when n is large.

Exercise 8.14: $e)\frac{x+1}{x} = 1 + \frac{1}{x}$. Not all of these integrals exist.

Exercise 8.16: Show that $\sum(f,P,S) \leq \sum(g,P,S)$ for every partition P of $[a,b]$ and every sample S for P.

Exercise 8.28: g is the sum of an integrable function and a spike function.

Exercise 8.32: f is not piecewise monotonic. It is easy to see that f is integrable on $[1,2]$. If you can show it is integrable on $[0,1]$then you are essentially done.

Exercise 8.34: b) $(b-a)^3/6$.

Exercise 8.41: For any partition P of $[0.1]$ you can find a sample S such that $\sum(R,P,S) = 0$.

Exercise 8.46: In equation 8.44, replace r by $\frac{1}{R}$, , and replace a and b by RA and RB.

Exercise 8.48: $\alpha(E_{ab}) = \pi ab$.

Exercise 8.50: If $a = 1/4$ then both areas are approximately 3.1416

Exercise 8.55: area $= 4\pi$.

Exercise 8.57: The areas are $5/12$ and 1.

Exercise 8.58: The area is $\frac{37}{12}$. Some fractions with large numerators may appear along the way.

Exercise 9.20: The last two formulas are obtained from the second by replacing t by $t/2$.

Exercise 9.29: I used exercise 9.28 with $x = \frac{\pi}{6}$ to find $\cos(\frac{\pi}{6})$. You can also give a more geometric proof.

Exercise 9.44: You will need to use (9.24).

Exercise 9.48: $\alpha(S_0^{\pi}(\sin)) = 2$.

Exercise 9.49: area $= \sqrt{2}$.

Exercise 9.69: $\int_0^{\pi/2} \sin(x)dx = 1$; $\int_0^{\pi/2} \sin^2(x)dx = \pi/4$; $\int_0^{\pi/2} \sin^4(x)dx = 3\pi/16$.

Exercise 10.25: $f'(a) = -\frac{1}{a^2}$.

Exercise 10.26: See example 10.9 and 9.26.

Exercise 10.27: $f'(a) = \frac{1}{(a+1)^2}$.

Exercise 10.28: $y = 2x - 4$; $y = -6x - 4$.

Exercise 11.6: I used formula 9.25

Exercise 11.15: $\frac{d}{dt}(|-100t|) = \frac{100t}{|t|}$.

Exercise 11.21: You can use the definition of derivative, or you can use the product rule and the reciprocal rule.

Exercise 11.24: $f'(x) = \ln(x)$, $g'(x) = \frac{ad-bc}{(cx+d)^2}$, $k'(x) = 2(2x+3)(x^2+3x+11)$

Exercise 11.29: $(g \circ (g \circ g))(x) = ((g \circ g) \circ g)(x) = x$ for $x \in \mathbf{R} \setminus 0, 1$. If you said $(f \circ f)(x) = x$, calculate both sides when $x = -1$.

Exercise 11.40: Use the definition of derivative. $h'(2) = 0$.

Exercise 11.43: $g'(x) = -\tan(x)$, $h'(x) = \tan(x)$, $k'(x) = \sec(x)$, $l'(x) = -\csc(x)$, $m'(x) = 9x^2\ln(5x)$, $n'(x) = \frac{\sqrt{x^2+1}}{x}$ (It requires a lot of calculation to simplify n'), $p'(x) = \frac{x^2}{x+4}$, $q'(x) = \sin(\ln(|6x|))$.

Exercise 12.14: d) Such a function k does exist.

Exercise 12.15: a) Use extreme value property.

Exercise 12.27: Proof is like given proof of corollary 12.26.

Exercise 12.31: Apply corollary 12.26 to $F - G$.

Exercise 12.35: Yes.

Exercise 12.36: You can apply the chain rule to the identity $f(-x) = f(x)$.

Exercise 13.14: The function to minimize is $f(x) =$distance$((0, \frac{9}{2}), (x, x^2))$.

Exercise 13.15: You may get a complicated equation of the form $\sqrt{f(x)} = \sqrt{g(x)}$ to solve. Square both sides and the equation should simplify.

Exercise 14.5: Apply the intermediate value property to $f - fp$.

Exercise 14.9: One of the zeros is in $[1, 2]$.

Exercise 14.10: I showed that if temp$(A) <$ temp$(B) <$ temp(D), then there is a point Q in $DC \cup CA$ such that temp$(Q) =$ temp(B).

Exercise 14.11: if temp$(A) <$ temp$(B) <$ temp$(C) <$ temp(D), find two points different from B that have the same temperature as B.

Exercise 14.17: You may want to define some of these functions using more than one formula.

Exercise 14.41: Use the extreme value property to get A and B.

Exercise 14.54: $f'(x) = 2\sqrt{a^2 - x^2}$; $h'(x) = \arccos(ax)$; $n'(x) = (a^2+b^2)e^{ax}\sin(bx)$; $p'(x) = a^3x^2e^{ax}$.

Exercise 14.55: It is not true that $l(x) = x$ for all x. Note that the image of l is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Exercise 15.5: $g^{(k)}(t) = tf^{(k)}(t) + kf^{(k-1)}(t)$.

Exercise 15.8: Use the antiderivative theorem twice.

Exercise 15.9: $(fg)^{(3)} = fg^{(3)} + 3f^{(1)}g^{(2)} + 3f^{(2)}g^{(1)} + f^{(3)}g$.

Exercise 15.13: You will need to use a few trigonometric identities, including the reflection law (9.18).

Exercise 15.22: $h(t) = h_0 + v_0t - \frac{1}{2}gt^2$.

Exercise 15.29: Use theorem 15.27 and corollary 12.26

Exercise 16.2: You probably will not be able to find a "single formula" for this. My function has a local maximum at $\frac{1}{2n+1}$ for all $n \in \mathbf{Z}^+$.

Exercise 16.8: The result is known if $p < q$. To get the result when $q < p$, apply 16.6 to f on $[q, p]$.

Exercise 16.13: Not all of these integrals make sense. $K'(x) = 1$ for all $x \in \mathbf{R}$. $L'(x) = 1$ for $x \in \mathbf{R}^+$. $L'(x) = -1$ for $x \in \mathbf{R}^-$. $L(0)$ is not defined.

Exercise 17.16: b) $-\ln(|\csc(e^x) + \cot(e^x)|)$; f) $\frac{1}{2}\ln(|\sin(2x)|)$. i) Cf example 9.68i.j) You did this in exercise 9.69.

Exercise 17.31: b) $\frac{1}{2}e^x(\sin(x) - \cos(x))$. When you do the second integration by parts, be careful not to undo the first. c) $x \arctan(x) - \frac{1}{2}\ln(1 + x^2)$. Let $g'(x) = 1$. d) and e) can be done easily without using integration by parts. f) If $r = -1$ the answer is $\frac{1}{2}(\ln(|x|))^2$.

Exercise 17.42: c) Let $u = \sqrt{x}$. You will need an integration by parts. d) Let $u = \ln(3x)$. e) Remember the definition of 2^x.

Exercise 17.49: a) $\frac{1}{2}a^2 \arcsin(\frac{x}{a}) + \frac{1}{2}x\sqrt{a^2 - x^2}$. If you forget how to find $\int \cos^2(\theta)d\theta$, review example 9.53. Also recall that $\sin(2x) = 2\sin(x)\cos(x)$. b) $\ln(\frac{x+\sqrt{a^2+x^2}}{a})$. c) and d) do not require a trigonometric substitution.

Exercise 17.53: a) $\frac{5}{4}$. b) $\frac{\pi}{6}$. c) $\frac{4}{15}$.

Exercise 17.54: $\frac{2\pi}{3} + \sqrt{3}$.

Exercise 17.64: (g) $\ln(\sqrt{x^2 + 2x + 2} + x + 1)$. First complete the square, and then reduce the problem to $\int \frac{1}{\sqrt{u^2+1}}du$.