

Math 111
Calculus I

R. Mayer

... there was far more imagination
in the head of Archimedes than in
that of Homer.

Voltaire[46, page 170]

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