Math 431 Group Quiz Friday, November 9, 2001

Please turn in at most one answer per problem to Cathy. Your main problems will be to organize your use of time efficiently, and to avoid inflicting undue emotional distress on your colleagues. Some of the problems (e.g., the second) might be too arithmetically involved to solve in 1+ hour, in which case you describe how you **could** solve it if you only had more time.

1. Find a nontrivial unit in $\mathbf{Q}(\sqrt{73})$.

2. Find integers x and y such that

$$2x^2 + 2xy + 3y^2 = 1607.$$

Explain your techniques. If you insist on a brute force search, please replace the RHS by $1607000 \cdots 1607$ (where there are fifty zeros in the middle of the decimal expansion).

3. Find the class number of $\mathbf{Q}(\sqrt[3]{6})$. (As you may have noticed in the class notes on the web, the Minkowski bound stated in class originally was for quadratic fields; the result for a field F of degree n, signature (r_1, r_2), and discriminant D_F is that every ideal contains a nonzero element α with

$$|N(\alpha)| \leq \frac{n!}{n^n} \left(\frac{4}{\pi}\right)^{r_2} \sqrt{|D_F|}\, N(I) \quad .)$$

4. Fill in the blank with (the possessive or adjectival form of) a proper name.

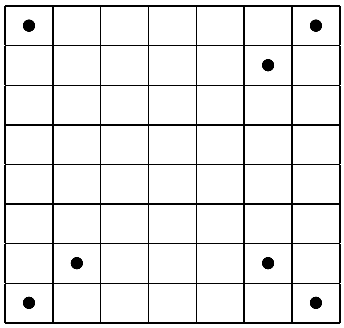
- _________ bound
- _________ Lemma
- _________ matrix
- _________ integers
- _________ bases
- _________ group
- _________ marginal remark
- _________ criterion
- _________ transformation

5. Prove that if α and β are elements of a number ring $\mathbf{Z}_F$ then there are x and y in $\mathbf{Z}_F$ such that
$$\alpha\beta = x\alpha^2 + y\beta^2.$$

6. Let u be an algebraic integer in a number field F of degree n such that all embeddings of u into the complex numbers have absolute value at most 1. Show that the embeddings all have absolute value equal to 1, and that u is a unit. Show that the coefficients in the minimal polynomial are bounded (depending on n, but independent of u). Show that u is a root of unity. (Hint: all of the powers of u satisfy the same conditions.)

7. (This problem is extra credit; however, it is worth 0 points. Solutions can be emailed to me; at the moment I don't know how to solve it.)

The 7 by 7 chessboard below has 7 PDR's on it. Find a sequence of moves that ends up with a PDR in the central square.



Definition: A PDR is a peaceful drunken rook that moves as a chess rook, except that once it starts moving it does not stop until it bumps into another piece, at which point it politely stops in the square before that piece. Thus if a PDR starts moving in a direction on a rank or file in which there are no other pieces, it disappears off the board, never to be seen again.