Note: This assignment has a second page! Include your **R** code and relevant output as part of your submission!

1. Let X_i $i = 1 \ldots n$, conditional on μ_i, have a Poisson(μ_i) distribution. Suppose that μ_i has a Gamma(α,β) distribution, where β is the rate parameter. (Think of this as a two stage process. First we generate a μ from the Gamma distribution, then, given μ, we generate a Poisson(μ) random variable.)

   (a) Find the marginal distribution for the Xs.

   (b) Write an **R** function to compute the log-likelihood $log(L(\alpha, \beta))$. Make sure that the arguments for your function are specified in the right order and form for the optimization function you plan to use!

   (c) Using the double expectation theorems, or otherwise, compute the expectation and variance of X. Thus compute the method of moments estimators for α and β. They will be useful as initial guesses for the coming optimization problem.

   (d) The following data are from a Poisson-Gamma mixture distribution. Find the MLE's for α and β. You will need to use one of the **R** optimization functions.

   5 10 5 10 14 16 7 11 7 4 6 19 6 9 14 8 3 7 5 11

   (e) Using the Hessian matrix from your optimizer, or from the hessian function of the numDeriv package, compute approximate SE's for the estimated parameters.

2. On January 28, 1986, Space Shuttle Challenger was launched in spite of warnings from the Morton-Thiokol engineers that the low temperature at launch might be a problem for the o-rings sealing the joints in the solid-fuel boosters. The Challenger exploded shortly after launch. There were 6 (primary) o-rings in the joints between boosters. Their function was to provide a tight seal at the joints, but allow the joints to flex. There had been 24 previous launches, all successful. However, there had been incidents of o-ring damage. Load the Shuttle Oring dataset with

```
ORings <- read.csv('http://people.reed.edu/~jones/Courses/Orings.csv')
```

The variables are *temp*, Y, and N. N is the number of o-rings, always 6. Y is the number of damaged o-rings after launch, and *temp* is the air temperature at launch in degrees Fahrenheit.

(a) Write an **R** function to compute the log-likelihood for the logistc regression model. Assume that observation Y_i has a Binomial$(6, P_i)$ distribution, where P is the probability of o-ring damage. The logistic regression model is:

$$\log\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 temp.$$

(b) Using one of the **R** optimization functions, maximize the log-likelihood for the logistic regression model. Display the output from the optimizer.

(c) Save the Hessian matrix; the diagonal elements of the inverse of the Hessian matrix are asymptotic variances for the coefficients. Compare your estimates and standard errors to the output of the glm function. To fit the logistic regression model with glm, use

```
glm.out <- glm(cbind(Y, 6-Y) ~ temp, data=ORings, family=binomial)
summary(glm.out)
```

(d) Use the fitted model to estimate the probability of o-ring damage at $32F$.