1. Show that if the design matrix $\mathbf{X}$ has a column of 1's (that is, the model includes an intercept term), then

$$\sum r_i = 0,$$

that is the residuals must sum to zero.

2. For the simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \qquad i = 1 \ldots n,$$

find the expression for the coefficients resulting from direct computation via the matrix formula

$$\hat{\beta} = (\mathbf{X}^t\mathbf{X})^{-1}\mathbf{X}^t\mathbf{Y}.$$

Note that the design matrix $\mathbf{X}$ has two columns: the first is all 1's, the second is the values $X_1, X_2, \ldots X_n$. Can you think of any reason not to do the calculation this way?

3. Suppose that $Y_i = \theta + \epsilon_i$, where ϵ_i are IID with $\mathbb{E}(\epsilon_i) = 0$ and finite variance. Find the least squares estimator for θ. Can you figure out what the minimum absolute deviation estimator for θ is? Proof not required. (The MAD estimator chooses θ to minimize $\sum |Y_i - \theta|$ rather than $\sum(Y_i - \theta)^2$. Experimenting with a small sample of numbers, say 5 or 6, may help!)

4. For each of the following distributions, try to find a conjugate or other convenient prior distribution. For that prior, produce an estimator such as the posterior mean, and find the posterior variance.

   (a) $X_1, X_2, \ldots X_n$ are *IID* Uniform(0,θ).

   (b) X is Negative Binomial(r,p). (r is known.)

   (c) $X_1, X_2, \ldots X_n$ are *IID* Poisson(λ).