Problem Set 8

Physics 442
Quantum Mechanics II

Due on Friday, April 5th, 2024

Problem 1

Using the variational principle with a Gaussian trial wave function,

$$\psi(x) = \psi_0 e^{-br^2}, \tag{1}$$

what is your best estimate of the ground state energy of hydrogen, and how does it compare with the actual ground state energy?

Problem 2

Griffiths & Schroeter Problem 8.4b — Using parity to ensure a first-excited state approximation.

Problem 3

Griffiths & Schroeter Problem 8.7 — stripping one electron from helium (ground state).

Problem 4

In situations where there are both bound and scattering states, completeness requires a sum over the bound states and an integral over the scattering states. The scattering states of the delta well, $V(x) = -\alpha\delta(x)$, look a lot like Fourier modes, and "any" function can be written in terms of its Fourier transform, so it would appear that any $f(x)$ could be written as a sum of the scattering states of the delta well. Show that the bound state cannot.

Problem 5

A "soft-sphere" is a sphere of radius R that has uniform potential energy U_0 inside of it (the potential energy outside the sphere is zero). A particle with energy E enters the soft sphere with impact parameter b, what is the scattering angle, θ_s, at which it emerges (assume $E > U_0$)?

Presentation Problem

This problem will either be presented in class on Friday (April 5th), or presented in written form, due by Friday. Take a look at it over the weekend, I'll ask for a volunteer on Monday. Whether or not you present the problem or not, you should solve it and be prepared to discuss it in class.

Problem 1*

For a massive photon, the Coulomb potential for a point charge is replaced by the Yukawa potential. For a proton with charge e and an electron with charge $-e$, the potential energy set up by the proton is

$$U(\mathbf{r}) = -\frac{e^2 e^{-\mu r}}{4\pi\epsilon_0 r} \tag{2}$$

with $\mu \equiv m_{\text{ph}} c/\hbar$ where m_{ph} is the photon mass. Estimate the ground state energy for "hydrogen" using this potential energy for the proton-electron interaction.