Problem Set 7

Physics 442
Quantum Mechanics II

Due on Friday, March 29th, 2024

Problem 1

In setting up perturbation theory, we expanded the energies and wavefunctions in a parameter λ, then collected equations in powers of λ and solved them individually. There is subtlety in the process – try it out in the context of a polynomial function:

For the polynomial, $f(x) = \epsilon x^2 + 2bx + c$ (for constants $b > 0$, $c > 0$, and small ϵ), find the roots. Now suppose that you want to use perturbation theory to approximate the roots, start with $x = x_0 + \epsilon x_1$ and use $f(x) = 0$ to collect equations in powers of ϵ and solve for x_0 and x_1. What roots do you get? You should be missing one – by expanding the actual roots in ϵ, explain why.

Problem 2

Griffiths & Schroeter Problem 7.21 – observable effects of fine structure.

Problem 3

Griffiths & Schroeter Problem 7.33 – finite size effect of the proton.

Problem 4

Griffiths & Schroeter Problem 8.2 – variational method practice.

Problem 5

Using the one-parameter trial wavefunction: $\psi(x) = Ax^p(a-x)$ (parameter p), find the best estimate for the ground state of the infinite square well (of width

a). Compare with the known ground state.

Problem 6

Suppose we have a potential energy $U(x) = -\alpha/x^2$ for constant α (positive), an attractive, "$1/r^2$" potential in one dimension. Show, from the time-independent Schrödinger's equation,

$$-\frac{\hbar^2}{2m}\psi''(x) - \frac{\alpha}{x^2}\psi(x) = E\psi(x),$$

with boundary condition $\psi(\pm\infty) = 0$, that there is no ground state (hint: Show that if you found a lowest energy state, with energy E_0, then rescaling the x coordinate via $y = \sigma x$ leads to a new, lower energy associated with the same state).

Presentation Problems

Here are some presentation problems (see syllabus for a description of the writeup/in-class presentation). These problems will either be presented in class on Friday (March 29th), or presented in written form, due by Friday. Take a look at them over the weekend, I'll ask for volunteers on Monday. Whether or not you present one of these problems, you should solve them and be prepared to discuss them in class.

Problem 1*

For an infinite square well (of width a) with a linear sloping bottom ($V(x) = \alpha x$ inside the well), find a bound on the ground state energy using the variational principle. What is the estimate that comes from first order time-independent perturbation theory?

Problem 2*

The variational principle gives us quite a bit of freedom in choosing the trial wave function, including random guessing. For the harmonic oscillator, in dimensionless form, given a $\bar{\psi}(q)$, we have

$$\langle E \rangle = \frac{1}{\sqrt{2}} \frac{\int_{-\infty}^{\infty} \left[-\bar{\psi}^* \bar{\psi}'' + \frac{1}{2} q^2 \bar{\psi}^* \bar{\psi} \right] dq}{\int_{-\infty}^{\infty} \bar{\psi}^* \bar{\psi} dq}. \tag{1}$$

If you make a grid with $q_j = j\Delta q$ for $j = -N_\infty \rightarrow N_\infty$, and let $\bar{\psi}_j \equiv \bar{\psi}(q_j)$, then we can approximate the above via (assuming we take $\bar{\psi}$ to be real):

$$\langle E \rangle = \frac{1}{\sqrt{2}} \frac{\sum_{j=-N_\infty}^{N_\infty} \left[-\frac{\bar{\psi}_{j+1} - 2\bar{\psi}_j + \bar{\psi}_{j-1}}{\Delta q^2} \bar{\psi}_j + \frac{1}{2} q_j^2 \bar{\psi}_j^2 \right] \Delta q}{\sum_{j=-N_\infty}^{N_\infty} \bar{\psi}_j^2 \Delta q}. \tag{2}$$

Given an initial set of values $\{\bar{\psi}_j\}_{j=-N_\infty}^{N_\infty}$, devise an update scheme (people use randomness) to try to minimize $\langle E \rangle$. In this dimensionless setting, the ground state energy of the harmonic oscillator is $1/2$.

Problem 3*

Find the (first order) relativistic perturbation of the energies for a one-dimensional harmonic oscillator.

Problem 4*

Working by analogy with rotations, find the Lorentz boost associated with an arbitrary direction velocity vector $\mathbf{v}$ (i.e. generate a boost in the $\hat{\mathbf{v}}$ direction with speed v).