Problem Set 4

Physics 442
Quantum Mechanics II

Due on Friday, February 23rd, 2024

Problem 1

Show that any one-dimensional function $f(x)$ can be decomposed into the eigenfunctions of the parity operator $\hat{\Pi}$.

Problem 2

Using $\hat{R}_z(\varphi)$ from (6.29), compute the transformed angular momentum operator:

$$\hat{\hat{\mathbf{L}}} = \hat{R}_z^{\dagger}\hat{\mathbf{L}}\hat{R}_z, \tag{1}$$

explicitly, working with the infinitesimal form of the rotation.

Problem 3

Griffiths & Schroeter Problem 6.13.

Problem 4

Griffiths & Schroeter Problem 6.19.

Problem 5

Griffiths & Schroeter Problem 6.22a (you've already established (6.51) in the previous problem, skip that one).

Problem 6

Griffiths & Schroeter Problem 6.18.

Problem 7

For an electrostatic potential $V(x)$ (working in one dimension for this problem), we can always add a constant offset without changing the resulting field — this represents a type of "gauge choice." For the infinite square well (use one that extends from 0 to a), find the effect of adding $qV(t)$ to the potential inside the well (a uniform, but time-varying potential "floor" for the well) by writing out the general time-dependent solution $\Psi(x,t)$.

Presentation Problems

These problems will either be presented in class on Friday (February 23rd), or presented in written form, due by Friday. Take a look at them over the weekend, I'll ask for volunteers on Monday. Whether or not you present one of these problems or not, you should solve them and be prepared to discuss them in class.

Problem 1*

You showed that there is a complex number u that can be associated with points from the upper (unit) hemisphere (Problem Set 3, Problem 4). In order to allow for a mapping that includes the north pole (and the lower hemisphere, as well), take $u = p/q$ for complex p and q (and with $p = 1$, $q = 0$ associated with the north pole). Work out the relations between $\{p, q\}$ and $\{x, y, z\}$.

The numbers p and q can be embedded in a vector, defining the "spinor" $\chi = (p \; q)^T$. For a rotation of the Cartesian coordinates about the z axis: $\bar{\mathbf{x}} = \mathbb{R}_z \mathbf{x}$, what is the matrix $\mathbb{S}_z$ that appropriately rotates the spinner χ?

Problem 2*

One way to make classical mechanics "look" more like quantum mechanics is to think about the time-independent probability density associated with particle motion. In one dimension, for a particle traveling along $x(t)$, we have $\rho(x,t) = \delta(x - x(t))$. For motion with period T, so that $x(t+T) = x(t)$, define the time average probability density:

$$\rho(x) \equiv \frac{1}{T/2} \int_0^{T/2} \rho(x,t) dt. \tag{2}$$

Write out the expression for $\rho(x)$ for an oscillatory particle of energy E moving under the influence of a potential energy $U(x)$. Check your expression with some examples ("infinite square well" and harmonic oscillator are good).