Problem Set 11

Physics 442
Quantum Mechanics II

Due on Friday, April 26th, 2024

Problem 1

For a perturbation with $H'_{aa} = H'_{bb} = 0$ and $H'_{ab} = H'_{ba} = \alpha e^{-t/\tau}$, if the particle starts in the a state at time $t = 0$, find the probability of transition to the b state as a function of time using first order perturbation theory. What is the probability in the limit $t \to \infty$?

Problem 2

Griffiths & Schroeter Problem 11.11 — Alternate "derivation" of spontaneous emission.

Problem 3

Griffiths & Schroeter Problem 11.24a-c — time-dependent perturbation theory without the two-level assumption. Just do parts a-c.

Problem 4

Griffiths & Schroeter Problem 11.27 — A perturbation for the infinite square well. Use your result from the previous problem. In addition to calculating the probability of transition to the first excited state, find the probability of transition to the state $n = 10$. What is the ratio of the magnitudes of these probabilities: $P_{10}/P_2 =$?

Presentation Problems

These problems will either be presented in class on Wednesday, April 24th, or presented in written form, due by Wednesday. Take a look at them, I'll ask for volunteers on Friday, April 19th. Whether or not you present one of these problems or not, you should solve them and be prepared to discuss them in class.

Problem 1*

Noether's theorem tells us that when a Lagrangian is unchanged by a transformation, there is an associated conservation law ("symmetry implies conservation").

If a Lagrangian depends on both Ψ and Ψ^*, and you find the perturbation $\Psi \to \Psi + \delta\Psi$ does not induce any change in the Lagrangian (to first order in $\delta\Psi$), then the conservation law $\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J}$ has

$$\begin{aligned}\rho &= \frac{\partial \mathcal{L}}{\partial \Psi_t}\delta\Psi + \frac{\partial \mathcal{L}}{\partial \Psi_t^*}\delta\Psi^* \\ J^x &= \frac{\partial \mathcal{L}}{\partial \Psi_x}\delta\Psi + \frac{\partial \mathcal{L}}{\partial \Psi_x^*}\delta\Psi^*\end{aligned} \tag{1}$$

(and similar expressions for J^y and J^z). For the Klein-Gordon Lagrangian (with electric potential V, so that qV is the potential energy):

$$\mathcal{L} = -\frac{1}{c^2}\Psi_t\Psi_t^* + \Psi_x\Psi_x^* - \frac{iq}{\hbar c^2}V(\Psi\Psi_t^* - \Psi^*\Psi_t) - \frac{q^2}{h^2c^2}V^2\Psi^*\Psi + \left(\frac{mc}{\hbar}\right)^2\Psi^*\Psi, \tag{2}$$

identify a transformation for Ψ that leaves $\mathcal{L}$ unchanged, and find the conserved ρ and current $\mathbf{J}$.

Problem 2*

For the differential equation,

$$\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & U(t) \\ U(t) & 0 \end{pmatrix}}_{\equiv \mathbb{H}(t)} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}, \tag{3}$$

with $a(0) = a_0$ and $b(0) = b_0$, show that the solution is

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = e^{\int_0^t \mathbb{H}(\bar{t})d\bar{t}} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}. \tag{4}$$

What happens if you try to solve

$$\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & U(t) \\ W(t) & 0 \end{pmatrix}}_{\equiv \mathbb{H}(t)} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \tag{5}$$

with the same method? (in this problem, $U(t)$ and $W(t)$ are functions of time, but otherwise arbitrary).

Problem 3*

Using the Crank-Nicolson method (available from the course website), take an infinite square well with unit length, and use a time-varying potential relevant for electromagnetic radiation, $U = -U_0 x \cos(\omega t)$. Start with a particle in the fifth excited state of the square well, pick a value for ω, and time-evolve the state numerically. How is the state $\Psi(x, t)$ distributed among the eigenstates of the square well as time goes on?