I

Schrödinger-Poisson System

A charge moves in the presence of an electric potential:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + qV\psi = i\hbar\frac{\partial \psi}{\partial t}$$

w/ Φ satisfying:

$\nabla^2 V = -\rho/\epsilon_0$ $\quad$ ρ is charge density.

Hydrogen is an example, w/ $\rho = q\delta^3(\vec{r})$, the point-proton at the origin.

We understand $\psi^*\psi$ to be a probability density – what, then, is

$$q\psi^*\psi \;\ldots ?$$

The charge itself forms a "cloud" – $\rho_q = q\psi^*\psi$

Shouldn't we include this in the interaction?

Focus on the charge q – the suggested starting point is:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + qV\psi = i\hbar\frac{\partial \psi}{\partial t} \qquad (*)$$

$\nabla^2 V = -\frac{1}{\epsilon_0} q\psi^*\psi$ – we can write the formal solution to this eqn. as:

$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\int_{\text{all space}} \frac{\psi^*(\vec{r}\,')\psi(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

and then $(*)$ becomes the nonlinear (& confusing):

$$-\frac{\hbar^2}{2m}\nabla^2\psi + q^2\psi\int_{\text{all space}} \frac{\psi^*(\vec{r}\,')\psi(\vec{r}\,')}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|}\,d\tau' = i\hbar\frac{\partial \psi}{\partial t}$$

What if we do the same thing w/ a gravitational self-interaction?

$$-\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi = i\hbar\frac{\partial \psi}{\partial t}$$

w/ $\nabla^2\Phi = 4\pi G\underbrace{m\psi^*\psi}_{\rho_m}$

- for Φ small – QM free-spreading dominates

- for Φ large – gravitational collapse dominates

On the cusp – an unstable equilibrium – if you perturb the system ... by measurement, it collapses (Penrose)

Issues: 1. QM

2. Need a t-dep., rel. theory of grav. (GR)

3. Need a t-dep. rel. theory of QM (K-G, Dirac, ⋮)