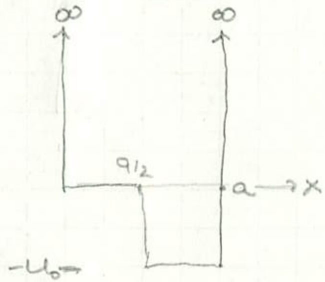
Dimensionless Model Problem

$$U(x) = \begin{cases} \infty & x<0,\ x>a \\ 0 & 0<x<a/2 \\ -U_0 & a/2<x<a \end{cases}$$



we want to solve:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

w/ $\psi(0) = 0 = \psi(a)$. Let $x = aX$ for dimless X:

$$-\frac{d^2\psi}{dX^2} + \underbrace{\frac{2ma^2}{\hbar^2}U}_{\equiv \tilde{U}}\psi = \underbrace{\frac{2ma^2}{\hbar^2}E}_{\equiv \tilde{E}}\psi$$

w/ $\tilde{U}(X) = \begin{cases} \infty & X<0,\ X>1 \\ 0 & 0<X<1/2 \\ -\tilde{U}_0 & 1/2<X<1 \end{cases}$

and $\psi(0) = 0 = \psi(1)$.

Our "residual" energy measure is now:

$$r_{\tilde{E}} = \tilde{E} - (j^2\pi^2 - \tfrac{1}{2}\tilde{U}_0).$$

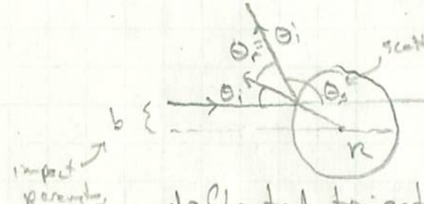
from methods

(show comparisons)

Scattering

We'll use a hard-sphere as an example to set the notation:

particle comes in, bounces off the hard sphere. Where does it end up?



the height above the center is the "impact parameter" – the angle between the initial direction & the end, deflected trajectory is the "scattering angle."

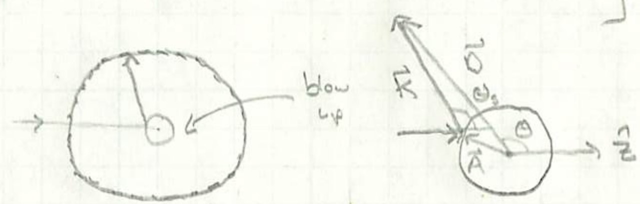
For our hard sphere, we have:

$$\sin\Theta_i = b/R$$

and

$$\Theta_s + 2\Theta_i = \pi \Rightarrow \Theta_s = \begin{cases} \pi - 2\sin^{-1}(b/R) & b<R \\ 0 & b>R \text{ (miss)} \end{cases}$$

If we have a spherical "particle detector" enclosing the experiment:



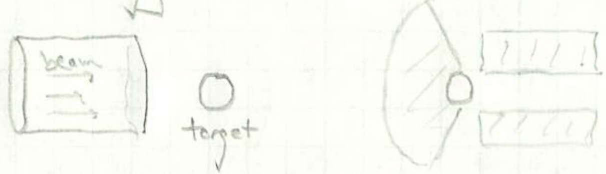
the scattering angle Θ_s is not the same as the polar angle Θ,

$$\hat{K}\cdot\hat{z} = \cos\Theta_s \qquad \hat{D}\cdot\hat{z} = \cos\Theta$$

But if the detector is far enough away, $\vec{D} = \vec{A} + \vec{K} \approx \vec{K}$ so $\Theta \approx \Theta_s$.

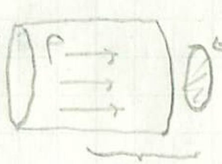
Particle Beam

Send in a large number of particles w/ the same energy.



the resulting angular distribution at the detector gives information about the particle-target interaction.

A uniform beam of particles travelling at speed v:



ρ is #/vol.
v is speed

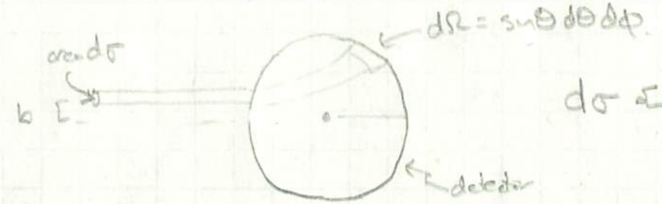
$v\Delta t$

The number of particles passing through a disk w/ area A in time Δt is:

$$N = \rho \cdot A \cdot v\Delta t \Rightarrow \frac{N}{A\,\Delta t} = \rho v = \mathcal{L}$$

"luminosity"

We know that particles in the vicinity of b will scatter into the vicinity of $\theta \approx \theta_s$



$$d\sigma \propto d\Omega$$

call the proportionality constant: $d\sigma = D(\theta)\, d\Omega$ (assuming spherical symmetry, else $D(\theta,\phi)$)

$$d\sigma = b\,db\,d\phi$$

→ $db = \left|\frac{db}{d\theta}\right| d\theta$, so

$$d\sigma = \frac{b\left|\frac{db}{d\theta}\right|}{\sin\theta} \underbrace{\sin\theta\, d\theta\, d\phi}_{= d\Omega} \quad \text{so} \quad D(\theta) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

"differential scattering cross section"

For a hard sphere, w/ $b = R\sin\left(\frac{\pi-\theta}{2}\right) = R\cos(\theta/2)$

$$\frac{db}{d\theta} = -\frac{R}{2}\sin(\theta/2)$$

→ $$D(\theta) = \frac{\frac{1}{2}R^2 \sin(\theta/2)\cos(\theta/2)}{\sin\theta} = R^2/4$$

If the luminosity of the outgoing beam is $\mathcal{L}'$, then

$$\underbrace{\mathcal{L}\, d\sigma}_{\text{rate of particles entering}} = \underbrace{\mathcal{L}' \cdot r^2 d\Omega}_{\text{rate of particles exiting}} \quad \text{so} \quad d\sigma = \underbrace{\frac{\mathcal{L}' r^2}{\mathcal{L}}}_{\text{"}D(\theta)\text{"}} d\Omega$$

→ $D(\theta)$ is, roughly, the rate of particles scattering into $da = r^2 d\Omega$.

the "total" cross section is the integral:

$$d\sigma = D(\theta)\,d\Omega \Rightarrow \sigma = \int_0^{2\pi}\int_0^{\pi} D(\theta)\sin\theta\,d\theta\,d\phi$$
$$= \int_0^{2\pi}\int_0^{\pi} b\left|\frac{db}{d\theta}\right| d\theta\,d\phi$$

for our hard spheres, we have:

$$\sigma = \int_0^{2\pi}\int_0^{\pi} R^2/4\,\sin\theta\,d\theta\,d\phi$$
$$= \pi R^2 \checkmark \qquad \text{(makes sense).}$$

<u>Rutherford</u> <u>Experiment</u>

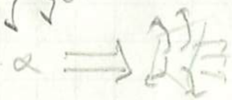
Thomson's "plum pudding" model for atoms - positively charged medium w/ e^- floating in it.

Rutherford (Marsden & Geiger) used high energy α-particles as the "beam" & gold foil as the target.

He expected

← small deflection

what they got



some large deflections - suggests a nucleus.