Problem 1 (11.3)

$I(t) = \frac{dq(t)}{dt} = -\omega q \sin(\omega t)$ & the power loss to Joule heating is: $P_J = I^2 R$

so $P_J = \omega^2 q^2 \sin^2(\omega t) R$ - if we time average, $P_J = \frac{1}{2} q^2 \omega^2 R$

the radiated power is: $P_R = \frac{\mu_0 q^2 d^2}{12\pi c}\omega^4$.

Setting the two equal, & solving for R gives: $\frac{\mu_0 q^2 d^2}{12\pi c}\omega^4 = \frac{1}{2} q^2 \omega^2 R$

or $R = \frac{\mu_0}{6\pi}\frac{\omega^2}{c} d^2$ & $\omega = 2\pi f$, $\lambda = cT = c/f \Rightarrow \omega = 2\pi c/\lambda$

$$= \underbrace{\frac{\mu_0 c \cdot 4\pi}{6}}_{\approx 80\pi^2}\left(\frac{d}{\lambda}\right)^2 \approx 790 (d/\lambda)^2\ \Omega$$

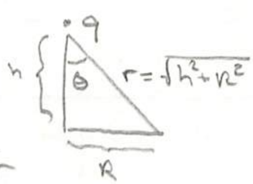
for $d = .05\,m$, & $\lambda \approx 10^3\,m$ (AM radio wavelength), $(d/\lambda)^2 = 2.5\times10^{-9}$
& $R = 2\times10^{-6}\,\Omega$, small.

Problem 2 (11.22)

a. $\vec{p}(t) = qd\cos(\omega t)\hat{z}$ w/ $\omega \equiv \sqrt{k/m}$, & setting the origin at the eq. location.

The intensity is $\vec{I} = \frac{\mu_0 (qd)^2 \omega^4}{32\pi^2 c}\frac{\sin^2\theta}{r^2}\hat{r}$ (11.21)

Then for $\sin^2\theta = \frac{R^2}{r^2}$, $\cos\theta = h/r$



$$I = \frac{\mu_0 (qd)^2 \omega^4}{32\pi^2 c}\frac{R^2}{(h^2+R^2)^2}\underbrace{\hat{r}\cdot\hat{z}}_{=\cos\theta}$$ floor normal

$$= \frac{\mu_0 (qd)^2 \omega^4}{32\pi^2 c}\frac{hR^2}{(h^2+R^2)^{5/2}}$$

To find the R that gives the maximum I, take $\frac{dI}{dR} = 0$

$$\frac{dI}{dR} = \frac{2\mu_0 (qd)^2\omega^4 hR}{32\pi^2 c (h^2+R^2)^{5/2}} - \frac{5/2\,\mu_0 (qd)^2 \omega^4 hR^2}{32\pi^2 c (h^2+R^2)^{7/2}} 2R = 0$$

$$= 2(h^2+R^2)R - 5R^3 = 0 \Rightarrow 2h^2 - 3R^2 = 0 \Rightarrow R = \sqrt{2/3}\, h$$

b. The total power going through the floor is: $P = \int_0^\infty \int_0^{2\pi} I \cdot R\,d\phi\,dR$

$$= \frac{\mu_0 (qd)^2 \omega^4 h}{16\pi c}\underbrace{\int_0^\infty \frac{R^3}{(h^2+R^2)^{5/2}}dR}_{=\frac{2}{3h}}$$

total radiated power here is

$P_{tot} = \frac{\mu_0 (qd)^2 \omega^4}{12\pi c}$ - 1/2 hits the floor

the other half hits the ceiling.

$$= \frac{\mu_0 (qd)^2 \omega^4}{24\pi c}$$

Problem 2 (continued)

Using the Landau-Lifschitz form of the radiation reaction force, for "external" force $F = -kx$, we have:

$$F_{LL} = \tau \dot{F} = -k\tau\dot{x} \quad w/ \ \tau = \frac{\mu_0 q^2}{6\pi m c}.$$

Then Newton's 2^{nd} law is $m\ddot{x} = -kx - k\tau\dot{x}$ - putting $x = e^{\alpha t}$ in, we get

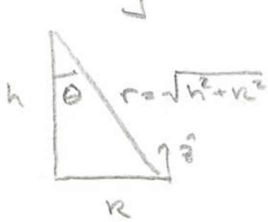
$$m\alpha^2 + k\tau\alpha + k = 0 \Rightarrow \alpha = \frac{-k\tau \pm \sqrt{(k\tau)^2 - 4km}}{2m}$$

& the exponential decay piece of the solution is $e^{-\frac{k\tau}{2m}t}$, this will reach e^{-1} at

$$\tau = \frac{2m}{k\tau} = \frac{12\pi m^2 c}{\mu_0 q^2 k}$$

Problem 3 (11.23)

a. For magnetic dipole radiation, $\bar{I} = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \frac{\sin^2\theta}{r^2} \hat{r}$ (11.39)



$\sin\theta = R/r$
$\cos\theta = h/r$

$$I = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \frac{R^2}{(h^2+R^2)^2} = \frac{3P}{8\pi} \frac{R^2}{(h^2+R^2)^2} \qquad P = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$

b. The maximum I occurs at $\frac{dI}{dR} = 0 = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3}\left(\frac{2R}{(h^2+R^2)^2} - \frac{2R^2 2R}{(h^2+R^2)^3}\right) = 0$

or $2R(h^2+R^2) - 4R^3 = 0 \Rightarrow 2h^2 - 2R^2 = 0 \Rightarrow R = h$

where the intensity is:

$$I_{max} = \frac{3P}{32\pi h^2}.$$

c. For $P = 35000$ W, $\omega = 90 \times 10^6$ 1/s, $h = 200$ m, $I \approx .026$ W/m²

$$I \approx .026 \frac{W}{m^2}\left(\frac{1m}{100cm}\right)^2\left(\frac{1000000\,\mu W}{1W}\right) \approx 2.6\ \mu W/cm^2 \text{ - no violation}$$

Problem 4 (11.19)

a. For $a(t) = \tau\dot{a}(t) + \frac{F(t)}{m}$, integrating in t from $t-\epsilon \to t+\epsilon$ gives

$$\int_{t-\epsilon}^{t+\epsilon} a(\bar{t})\,d\bar{t} = \int_{t-\epsilon}^{t+\epsilon} \tau\,\dot{a}(\bar{t})\,d\bar{t} + \int_{t-\epsilon}^{t+\epsilon} \frac{F(\bar{t})}{m}\,d\bar{t}$$

$$\int_{t-\epsilon}^{t+\epsilon} \frac{dv(\bar{t})}{d\bar{t}}\,d\bar{t} = \tau\left(a(t+\epsilon) - a(t-\epsilon)\right) + \int_{t-\epsilon}^{t+\epsilon} F(\bar{t})\,d\bar{t}$$

$$\underbrace{v(t+\epsilon) - v(t-\epsilon)}_{\to 0 \text{ as } \epsilon\to 0,\ \text{even if } a \text{ is discont.},} = \tau\left(a(t+\epsilon) - a(t-\epsilon)\right) + \underbrace{\int_{t-\epsilon}^{t+\epsilon} F(\bar{t})\,d\bar{t}}_{\to 0 \text{ as } \epsilon\to 0,\ \text{even if discontinuous } F.}$$

so the limit of this eqn. is $\lim_{\epsilon\to 0}\left(a(t+\epsilon) - a(t-\epsilon)\right) = 0$

b. In i. $a(t) = \tau\dot{a}(t) \Rightarrow a_i(t) = Ae^{t/\tau}$

in ii $a(t) = \tau\dot{a}(t) + \underbrace{F/m}_{=f}$ has $\dot{a} = \frac{1}{\tau}(a - f)$ let $u = a - f$, then $\dot{u} = \frac{1}{\tau}u \Rightarrow u = Be^{t/\tau}$

so $a_{ii}(t) = u(t) + f = Be^{t/\tau} + f$

in iii $a(t) = \tau\dot{a}(t) \Rightarrow a_{iii}(t) = Ce^{t/\tau}$

c. at $t=0$, we have $a_i(0) = A = a_{ii}(0) = B + f$, & at $t = T$,

$$a_{ii}(T) = Be^{T/\tau} + f = a_{iii}(T) = Ce^{T/\tau}$$

eliminating B using $B = A - f$, we get $(A - f)e^{T/\tau} + f = Ce^{T/\tau}$, or

$$C = A - f(1 - e^{-T/\tau}).$$

setting $C = 0$ eliminates the runaway solution in iii, but $A = \frac{F}{m}(1 - e^{-T/\tau}) \neq 0$, so we have pre-acceleration. Setting $A = 0$ eliminates the preacceleration, but $C = -f(1 - e^{-T/\tau}) \neq 0$, so we get the runaway behavior in iii.

d Taking $C = 0$, we have $A = f(1 - e^{-T/\tau})$ & $B = A - f = -fe^{-T/\tau}$, then

$$a_i(t) = \frac{F}{m}(1 - e^{-T/\tau})e^{t/\tau},\quad a_{ii}(t) = \frac{F}{m}\left(1 - e^{(t-T)/\tau}\right),\quad a_{iii}(t) = 0$$

w/

$$v_i(t) = \int a_i\,dt = \frac{F\tau}{m}(1 - e^{-T/\tau})e^{t/\tau}\ \ (v_i \xrightarrow[t\to-\infty]{} 0),\quad v_{ii}(t) = \frac{F}{m}t - \frac{F\tau}{m}e^{(t-T)/\tau} + \underbrace{J}_{\text{const.}}$$

& $v_i(0) = v_{ii}(0) \Rightarrow \frac{F\tau}{m}(1 - e^{-T/\tau}) = -\frac{F\tau}{m}e^{-T/\tau} + J \Rightarrow J = F\tau/m$

so

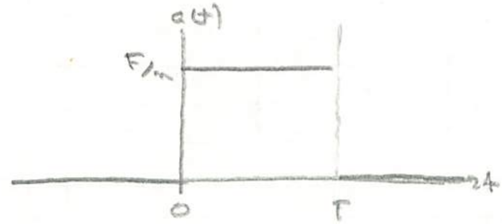
$$v_{ii}(t) = F/m\left(t + \tau - \tau e^{(t-T)/\tau}\right),\quad v_{iii}(t) = K \leftarrow \text{const}$$

w/

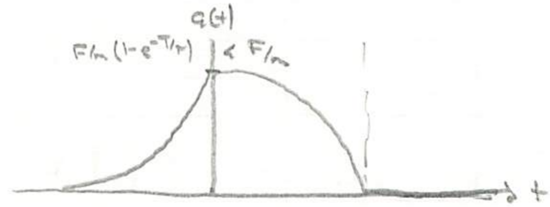
$$v_{ii}(T) = F/m\,T = v_{iii}(T) = K,\ \ \&\ \ v_{iii}(t) = \frac{FT}{m}.$$

Problem 4 (continued)

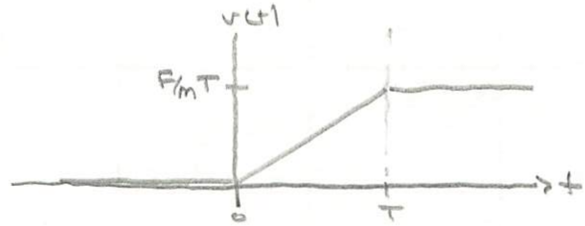
For an uncharged particle



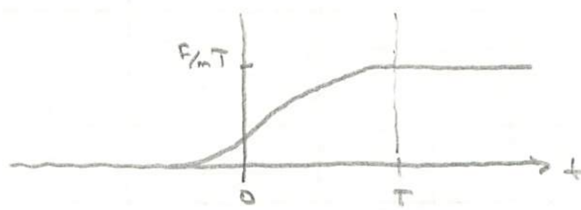
For a charged particle:



For an uncharged particle



For a charged particle



Problem 5

A particle moving w/ constant acceleration does radiate according to the Larmor formula, $P = \frac{\mu_0 q^2 a^2}{6\pi c} \neq 0$

It does not experience a radiation reaction force $F_r = \frac{\mu_0 q^2 \dot{a}}{6\pi c} = 0$

Problem 6

The time constant $\tau = \frac{\mu_0 q^2}{6\pi m c}$, for an e^- w/ $m = 9.11\times10^{-31}$ kg, $q = -1.6\times10^{-19}$ C is $\tau \simeq 6.2\times10^{-24}$ s.

Functions

```
In[1]:= Poynter[x_, y_, z_, t_] := Module[{tr, T, u, rvec, sr, srhat, pref, Ee, Bb, Sout},
    pref = 1.0;
    rvec = {x, y, z};
    tr = T /.  FindRoot[c (t - T) - Sqrt[(rvec - w[T]).(rvec - w[T])], {T, t}];
    sr = rvec - w[tr];
    srhat = sr / Sqrt[sr.sr];
    u = c srhat - v[tr];
    Ee =
     pref Sqrt[sr.sr] / (u.sr)^3 ( (c^2 - v[tr].v[tr]) u + Cross[sr, Cross[u, a[tr]]]);
    Bb = 1 / c Cross[sr, Ee];
    Sout = Cross[Ee, Bb];
    Return[Sout];
   ]
```

Example 1

Set the trajectory, velocity and acceleration.

```
In[2]:= w[t_] := {0, 0, d Cos[wo t]}
```

```
In[3]:= v[t_] := {0, 0, -d wo Sin[wo t]}
```

```
In[4]:= a[t_] := {0, 0, -d wo^2 Cos[wo t]}
```

Set constants relevant to the trajectory.

```
In[5]:= c = 1.;
d = .01;
wo = 5;
T = 2 Pi / wo;
```

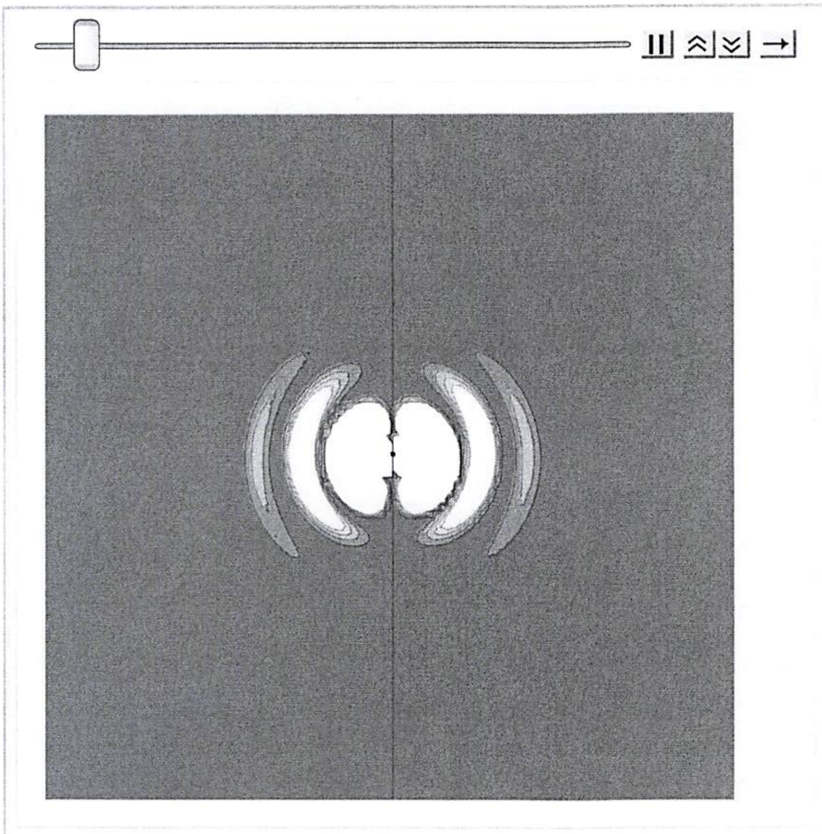
Calculate the fields in the y - z plane (for x = 0), make contour plots of the magnitude of the Poynting vector, and plot the motion of the particle :

```
In[9]:= stabvel = Table[ Table[{y, z, Poynter[0., y, z, t].Poynter[0., y, z, t]},
     {y, -5., 5., .1}, {z, -5., 5, .1}], {t, 0, 2 T, T / 10}];
veltab = Table[Partition[ Flatten[stabvel〚h〛], 3], {h, 1, Length[stabvel]}];
movvel = Table[ListContourPlot[veltab〚j〛,
     PlotRange → {{-5, 5}, {-5, 5}, {-.01, .01}}], {j, 1, Length[veltab]}];
charge = Table[Graphics[Point[{0, 5 d Cos[wo t]}],
     PlotRange → {{-5, 5}, {-5, 5}}], {t, 0, 2 T, T / 10}];
mov = Table[Show[charge〚j〛, movvel〚j〛], {j, 1, Length[charge]}];
```

Animate the flip - book.

In[14]:=
```
ListAnimate[mov]
```

Out[14]=



Example 2

Set the trajectory, velocity and acceleration.

In[15]:=
```
w[t_] := {0, d Sin[wo t], d Cos[wo t]}
```

In[16]:=
```
v[t_] := {0, d wo Cos[wo t], -d wo Sin[wo t]}
```

In[17]:=
```
a[t_] := {0, -d wo^2 Sin[wo t], -d wo^2 Cos[wo t]}
```

Set constants relevant to the trajectory.

In[18]:=
```
c = 1.;
d = .01;
wo = 5;
T = 2 Pi / wo;
```

Calculate the fields in the y - z plane (for x = 0), make contour plots of the magnitude of the electric field, and plot the motion of the particle :

```
In[22]:= stabvel = Table[ Table[{y, z, Poynter[0., y, z, t].Poynter[0., y, z, t]},
             {y, -5., 5., .1}, {z, -5., 5, .1}], {t, 0, 2 T, T / 10}];
         veltab = Table[Partition[ Flatten[stabvel[[h]]], 3], {h, 1, Length[stabvel]}];
         movvel = Table[ListContourPlot[veltab[[j]],
             PlotRange → {{-5, 5}, {-5, 5}, {-.01, .01}}], {j, 1, Length[veltab]}];
         charge = Table[Graphics[Point[{5 d Sin[wo t], 5 d Cos[wo t]}],
             PlotRange → {{-5, 5}, {-5, 5}}], {t, 0, 2 T, T / 10}];
         mov = Table[Show[charge[[j]], movvel[[j]]], {j, 1, Length[charge]}];
```

Animate the flip - book.

```
In[27]:= ListAnimate[mov]
Out[27]=
```