Problem 2 (11.12) | Problem Set 5

Using the Larmor power: $P = \frac{\mu_0 q^2 a^2}{6\pi c}$ w/ $a = 9.8\,\mathrm{m/s^2}$ (so that P is constant)

We have: $\Delta U = P \cdot \Delta t$, w/ $d = \frac{1}{2} g \Delta t^2 \Rightarrow U = P \cdot \sqrt{\frac{2d}{g}}$.

Setting $d = .01\,\mathrm{m}$, $q = 1.6\times10^{-19}\,\mathrm{C}$, $c = 3\times10^{8}\,\mathrm{m/s}$ & $\mu_0 = 4\pi\times10^{-7}\,\mathrm{N/A^2}$, we have

$$\Delta U = 2.47\times10^{-58}\,\mathrm{J}$$

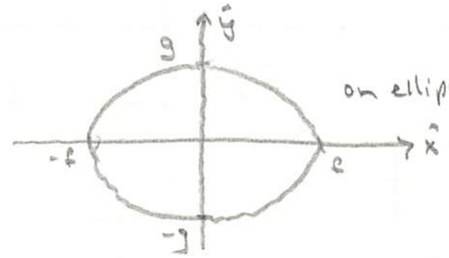
The initial potential energy is: $W = mgd$ w/ $m = 9.11\times10^{-31}\,\mathrm{kg}$, $W = 8.93\times10^{-32}\,\mathrm{J}$

so

$$\frac{\Delta U}{W} = 2.77\times10^{-22} \quad \text{(very little)}$$

Problem 3

$\vec{w}(t) = f\cos(\omega t)\hat{x} + g\sin(\omega t)\hat{y}$, sketch:



an ellipse.

$\vec{a}(t) = \ddot{\vec{w}}(t) = -\omega^2\left(f\cos(\omega t)\hat{x} + g\sin(\omega t)\hat{y}\right)$

w/

$$a^2 = \omega^4\left(f^2\cos^2(\omega t) + g^2\sin^2(\omega t)\right)$$

& the time-averaged Larmor-formula power is:

$$\langle P\rangle = \frac{\mu_0 q^2}{6\pi c}\langle a^2\rangle = \frac{\mu_0 q^2\omega^4(f^2+g^2)}{12\pi c}$$

for a circular trajectory: $\langle P_0\rangle = \frac{\mu_0 q^2\omega^4 f^2}{6\pi c}$
$(g = f)$

for the line trajectory $(g=0)$: $\langle P_1\rangle = \frac{\mu_0 q^2\omega^4 f^2}{12\pi c}$ so $\langle P_0\rangle = 2\langle P_1\rangle$, the circle radiates more. $\frac{\langle P_0\rangle}{\langle P_1\rangle} = 2$

Problem 4

$\vec{p}(t) = p_0\left(\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\right) \Rightarrow \ddot{\vec{p}}(t) = -\omega^2 p_0\left(\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\right)$

& $\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$, $\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$

w/ $\hat{r}\times\hat{x} = \cos\theta\cos\phi\,\hat{\phi} + \sin\phi\,\hat{\theta}$, $\hat{r}\times\hat{y} = \cos\theta\sin\phi\,\hat{\phi} - \cos\phi\,\hat{\theta}$

so

$$\vec{B}^{rad}(\vec{r},t) = +\frac{\mu_0}{4\pi r c}\omega^2 p_0\left[\cos(\omega t_0)\cos\theta\cos\phi\,\hat{\phi} + \cos(\omega t_0)\sin\phi\,\hat{\theta} + \sin(\omega t_0)\cos\theta\sin\phi\,\hat{\phi} - \sin(\omega t_0)\cos\phi\,\hat{\theta}\right]$$

$t_0 = t - r/c$

$$= \frac{\mu_0\omega^2 p_0}{4\pi r c}\left[\left(\cos(\omega t_0)\sin\phi - \sin(\omega t_0)\cos\phi\right)\hat{\theta} + \cos\theta\left(\cos(\omega t_0)\cos\phi + \sin(\omega t_0)\sin\phi\right)\hat{\phi}\right]$$

$$= \frac{\mu_0\omega^2 p_0}{4\pi r c}\left[\sin(\phi - \omega t_0)\hat{\theta} + \cos\theta\cos(\phi - \omega t_0)\hat{\phi}\right]$$

Problem 4 (continued)

$$\vec{E}^{rad}(\vec{r},t) = c\hat{r}\times\vec{B}^{rad}(\vec{r},t)$$

$$= \frac{\mu_0\omega^2}{4\pi r}p_0\left[\sin(\phi-\omega t_0)\hat{\phi} - \cos\theta\cos(\phi-\omega t_0)\hat{\theta}\right]$$

$$\to \vec{S}^{rad}(\vec{r},t) = \frac{1}{\mu_0}\vec{E}^{rad}(\vec{r},t)\times\vec{B}^{rad}(\vec{r},t)$$

$$= \frac{\mu_0}{c}\left(\frac{\omega^2 p_0}{4\pi r}\right)^2\left[\sin^2(\phi-\omega t_0)+\cos^2\theta\cos^2(\phi-\omega t_0)\right]\hat{r}$$

Problem 5

$\vec{w}(t) = \sqrt{b^2+(ct)^2}\,\hat{z}$ to find the retarded time at $t=0$, $\vec{r} = x\hat{x}+z\hat{z}$, we have to solve:

$$c^2(\underset{0}{\underset{\shortparallel}{t}}-t_r)^2 = x^2+\left(z-\sqrt{b^2+(ct_r)^2}\right)^2 \quad \text{for } t_r:$$

$$c^2t_r^2 = x^2+z^2-2z\sqrt{b^2+(ct_r)^2}+b^2+c^2t_r^2$$

or $$\sqrt{b^2+(ct_r)^2} = \frac{1}{2z}(x^2+z^2+b^2) \quad \text{squaring:}$$

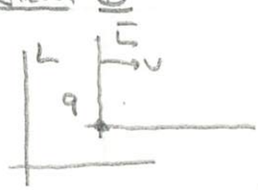
$$c^2t_r^2 = \frac{1}{4z^2}(x^2+z^2+b^2)^2-b^2$$

so $$t_r = -\frac{1}{c}\left[\frac{1}{4z^2}(x^2+z^2+b^2)^2-b^2\right]^{1/2}$$

taking the $-$ root so that $t_r < t$

As $z\to 0$, $t_r \to -\infty$.

Problem 6



In $\bar{L}$: $\bar{\rho} = q\delta(\bar{x})\delta(\bar{y})\delta(\bar{z})$

In L: $\rho = q\delta(x-vt)\delta(y)\delta(z)$

we have $\bar{x} = \gamma(x-vt)$, $\bar{y}=y$, $\bar{z}=z$, so

$$\bar{\rho} = q\delta(\gamma(x-vt))\delta(y)\delta(z) = \frac{1}{\gamma}q\delta(x-vt)\delta(y)\delta(z)$$

using $\delta(ku) = \frac{1}{|k|}\delta(u)$

or $\bar{\rho} = \frac{1}{\gamma}\rho \Rightarrow \rho = \gamma\bar{\rho}$.

Problem 1

Part a.

In[1]:=
```
w[t_] := d Cos[omega t] {0, 1, 0}
```

In[2]:=
```
r = {d / 100, -d / 10, 0};
```

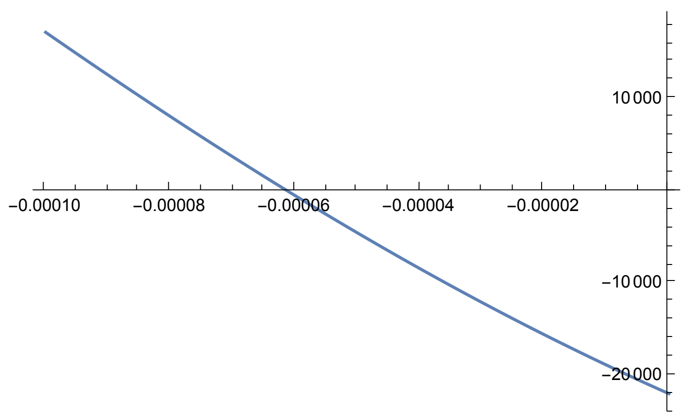
In[3]:=
```
t = 0;
```

In[4]:=
```
omega = 10 000;
d = 20 000;
c = 3 × 10 ^ 8;
```

In[7]:=
```
F[p_] := c (t - p) - Sqrt[(r - w[p]).(r - w[p])]
```

In[8]:=
```
Plot[F[p], {p, -10 ^ (-4), 0}]
```

Out[8]=



In[9]:=
```
tr = p /. FindRoot[F[p], {p, 0.}][[1]]
```

Out[9]= -0.0000612268

At time t_r, the particle is at location:

In[10]:=
```
w[tr]
```

Out[10]=

{0., 16 366.9, 0.}

Very far from where it is at time 0:

In[11]:=
```
w[0]
```

Out[11]=

{0, 20 000, 0}

Part b.

In[12]:=
```
w[t_] := d Cos[omega t] {0, 1, 0}
```

In[13]:=
```
r = {d / 100, -d / 10, 0};
```

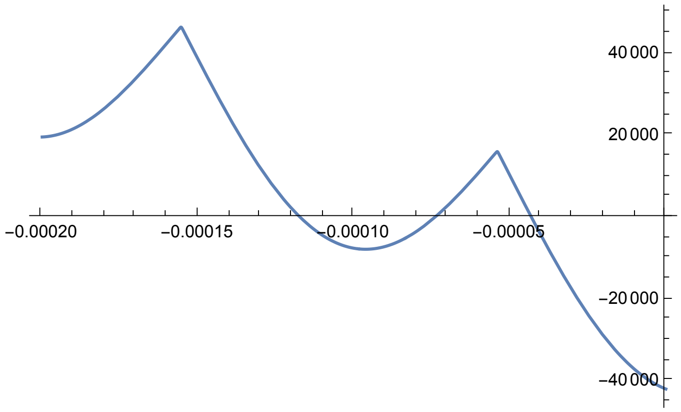
In[14]:=
```
t = 0;
```

In[15]:=
```
omega = 30 000;
d = 40 000;
c = 3 × 10 ^ 8;
```

In[18]:=
```
F[p_] := c (t - p) - Sqrt[(r - w[p]).(r - w[p])]
```

In[19]:=
```
Plot[F[p], {p, -.0002, 0}]
```

Out[19]=



The plot above has two roots, indicating that the particle achieves a speed greater than c:

In[20]:=
```
omega d - c
```

Out[20]=
```
900 000 000
```

Indeed, the maximum speed of the particle is significantly greater than c. The three roots are

In[21]:=
```
FindRoot[F[p], {p, -.00015}]
```

Out[21]=
```
{p → -0.000117294}
```

In[22]:=
```
FindRoot[F[p], {p, -.0003}]
```

Out[22]=
```
{p → -0.0000740259}
```

In[23]:=
```
FindRoot[F[p], {p, -.00005}]
```

Out[23]=
```
{p → -0.0000431282}
```