Problem 1 (10.15) | | Problem Set 4

$\vec{w}(t) = a(\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}) \qquad \vec{r} = z\hat{z}$

the retarded time is: $c^2(t-t_r)^2 = a^2+z^2 \Rightarrow t - t_r = \frac{1}{c}\sqrt{a^2+z^2} \Rightarrow t_r = t - \frac{1}{c}\sqrt{a^2+z^2}$

$\vec{\mathscr{r}} = \vec{r} - \vec{w}(t_r) = -a\cos(\omega t_r)\hat{x} - a\sin(\omega t_r)\hat{y} + z\hat{z}\ , \ \mathscr{r} = \sqrt{a^2+z^2}$

$\vec{v} = \dot{\vec{w}}(t_r) = a\omega(-\sin(\omega t_r)\hat{x} + \cos(\omega t_r)\hat{y})\ , \ \vec{\mathscr{r}}\cdot\vec{v} = 0$

Then $V(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\frac{1}{\mathscr{r} - \vec{\mathscr{r}}\cdot\vec{v}/c} = \frac{q}{4\pi\epsilon_0\sqrt{a^2+z^2}}$

$\vec{A}(\vec{r},t) = \frac{\mu_0 q\vec{v}}{4\pi}\frac{1}{\mathscr{r} - \vec{\mathscr{r}}\cdot\vec{v}/c} = \mu_0 q\frac{a\omega(-\sin(\omega t_r)\hat{x} + \cos(\omega t_r)\hat{y})}{4\pi\sqrt{a^2+z^2}}$

Problem 2

To compute $\vec{E}$, we need $\vec{a} = \ddot{\vec{w}}(t_r) = -a\omega^2(\cos(\omega t_r)\hat{x} + \sin(\omega t_r)\hat{y})$

$\vec{u} = c\hat{\mathscr{r}} - \vec{v}$ w/ $\vec{\mathscr{r}}\cdot\vec{u} = c\mathscr{r} - \underbrace{\vec{\mathscr{r}}\cdot\vec{v}}_{=0} = c\mathscr{r}$

$$\vec{\mathscr{r}}\times(\vec{u}\times\vec{a}) = \vec{u}(\vec{\mathscr{r}}\cdot\vec{a}) - \vec{a}(\vec{\mathscr{r}}\cdot\vec{u})$$
$$= \vec{u}(a^2\omega^2) - \vec{a}(c\mathscr{r})$$

we computed $\mathscr{r} = \sqrt{a^2+z^2}$ in the previous problem, then:

$$\vec{u} = \frac{c(-a\cos(\omega t_r)\hat{x} - a\sin(\omega t_r)\hat{y} + z\hat{z})}{\sqrt{a^2+z^2}} - a\omega(-\sin(\omega t_r)\hat{x} + \cos(\omega t_r)\hat{y})$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{u}\cdot\vec{\mathscr{r}})^3}\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{c^3\mathscr{r}^2}\left[(c^2 - v^2 + \overbrace{a^2\omega^2}^{=v^2})\vec{u} - \vec{a}(c\mathscr{r})\right] = \frac{\mu_0 q}{4\pi}\left[\frac{c\vec{u}}{\mathscr{r}^2} - \frac{\vec{a}}{\mathscr{r}}\right]$$

$$= \frac{\mu_0 q}{4\pi}\left[\frac{c}{a^2+z^2}\left(\left(-\frac{ac}{\sqrt{a^2+z^2}}\cos(\omega t_r) + a\omega\sin(\omega t_r)\right)\hat{x} + \left(\frac{-ac}{\sqrt{a^2+z^2}}\sin(\omega t_r) - a\omega\cos(\omega t_r)\right)\hat{y} + \frac{cz\hat{z}}{\sqrt{a^2+z^2}}\right)\right.$$
$$\left. + \frac{a\omega^2}{\sqrt{a^2+z^2}}\left(\cos(\omega t_r)\hat{x} + \sin(\omega t_r)\hat{y}\right)\right]$$

## Problem 4 (10.20)

For $\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{u}\cdot\vec{\mathscr{r}})^3}\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]$

w/ $\vec{w} = w(t)\hat{x}$, $\vec{r} = x\,\hat{x}$, $\vec{\mathscr{r}} = \vec{r} - \vec{w}(t_r) = (x - w(t_r))\hat{x}$, $\vec{v} = \dot{w}(t_r)\hat{x}$

$\vec{u} = c\hat{\mathscr{r}} - \vec{v} = (c-v)\hat{x}$, then $\vec{u}\cdot\vec{\mathscr{r}} = (c-v)\mathscr{r}$ + $\vec{\mathscr{r}}\times(\vec{u}\times\vec{a}) = 0$.

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{(c-v)^3\mathscr{r}^2}\left[(c^2-v^2)(c-v)\hat{x}\right] = \frac{q}{4\pi\epsilon_0\mathscr{r}^2}\frac{(c-v)(c+v)}{(c-v)(c-v)}\hat{x} = \frac{q}{4\pi\epsilon_0\mathscr{r}^2}\frac{c+v}{c-v}\hat{x}$$

If $x < w(t_r)$, $\vec{u} = c\hat{\mathscr{r}} - \vec{v} = -(c+v)\hat{x}$, w/ $\vec{\mathscr{r}} = -\mathscr{r}\hat{x}$, so $\vec{u}\cdot\vec{\mathscr{r}} = (c+v)\mathscr{r}$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{(c+v)^3\mathscr{r}^2}\left[-(c^2-v^2)(c+v)\hat{x}\right] = \frac{-q}{4\pi\epsilon_0\mathscr{r}^2}\frac{(c-v)(c+v)}{(c+v)(c+v)}\hat{x} = -\frac{q}{4\pi\epsilon_0\mathscr{r}^2}\frac{c-v}{c+v}\hat{x}$$

in both cases, $\vec{B} = \frac{1}{c}\hat{\mathscr{r}}\times\vec{E} \sim \hat{x}\times\hat{x} = 0$.

## Problem 5


$$\vec{E}(\vec{r},t) = \frac{q_2}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}\frac{(1-v^2/c^2)}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}} \qquad \theta = 0,\ \text{so } \sin\theta = 0,\ \vec{R} = -vt\,\hat{z}$$

$$= \frac{q_2}{4\pi\epsilon_0(vt)^2}(1-v^2/c^2)\,\hat{z}$$

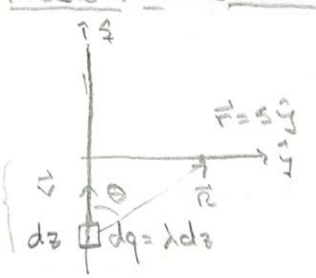
the force on $q_1$ is $\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0(vt)^2}(1-v^2/c^2)\,\hat{z}$

The force on $q_2$ due to $q_1$ is: $\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0(vt)^2}\,\hat{z}$

The forces are not equal + opposite ... Newton's third law is violated.

## Problem 6 (10.22)

a.




For the source "point" we have: $z = vt \Rightarrow dz = v\,dt$

$\vec{R} = -z\hat{z} + s\hat{y}$ w/ $R = \sqrt{z^2+s^2}$ + $R\sin\theta = s$

$$d\vec{E} = \frac{\lambda dz}{4\pi\epsilon_0 R^3}\frac{(1-v^2/c^2)}{(1-v^2\sin^2\theta/c^2)^{3/2}}\vec{R} \qquad \text{w/ } R^2 - \frac{v^2}{c^2}R^2\sin^2\theta = z^2 + s^2(1-v^2/c^2)$$

$$= \frac{\lambda dz}{4\pi\epsilon_0}\frac{(1-v^2/c^2)}{(z^2+s^2(1-v^2/c^2))^{3/2}}(-z\hat{z} + s\hat{y})$$

$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0}\left(1-v^2/c^2\right)\left[-\hat{z}\underbrace{\int_{-\infty}^{+\infty}\frac{z\,dz}{\left(z^2+(1-v^2/c^2)s^2\right)^{3/2}}}_{} + s\hat{y}\int_{-\infty}^{+\infty}\frac{dz}{\left(z^2+(1-v^2/c^2)s^2\right)^{3/2}}\right]$$

$$= \frac{-1/2}{\sqrt{z^2+(1-v^2/c^2)s^2}}\Bigg|_{z=-\infty}^{+\infty} = 0$$

For the other integral, note that $\frac{d}{dz}\left(\frac{\alpha z}{\sqrt{z^2+A^2}}\right) = \frac{\alpha}{\sqrt{z^2+A^2}} - \frac{\alpha z^2}{(z^2+A^2)^{3/2}}$

$$= \frac{\alpha A^2}{(z^2+A^2)^{3/2}}$$

take $\alpha = 1/A^2$, then

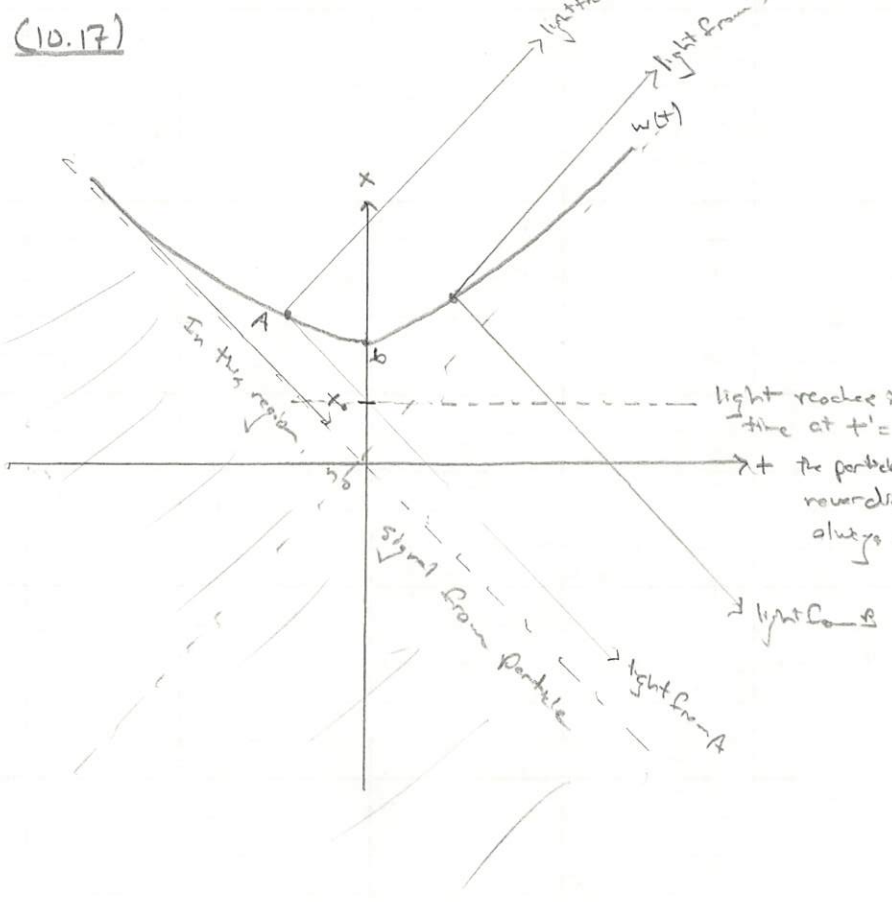
$$\vec{E} = \frac{\lambda(1-v^2/c^2)}{4\pi\epsilon_0}s\hat{y}\left[\frac{1}{(1-v^2/c^2)s^2}\cdot\frac{z}{\left(z^2+(1-v^2/c^2)s^2\right)^{1/2}}\Bigg|_{z=-\infty}^{+\infty}\right]$$

$$= \frac{\lambda}{2\pi\epsilon_0 s}\hat{y}_{\;s=d} \Rightarrow \vec{E} = \frac{\lambda}{2\pi\epsilon_0 d}\hat{s} \text{ in general}$$

locally $\cdot\ \hat{z}$

5. $\vec{B} = \frac{1}{c^2}\vec{v}\times\vec{E}$ w/ $\vec{v} = v\hat{z}$, $\vec{B} = \frac{\lambda v}{2\pi\epsilon_0 c^2}\frac{1}{d}\hat{\Phi} = \frac{\mu_0\lambda v}{2\pi d}\hat{\Phi}$.

Problem 3 (10.17)




light reaches $x_0$ for the first time at $t' = -x_0/c$, once the particle is "seen" it never disappears — there is always a light line crossing $x_0$ after $t' = -x_0/c$.