Problem 1

For $\cosh A \equiv \frac{1}{2}(e^{A}+e^{-A})$ & $\sinh A = \frac{1}{2}(e^{A}-e^{-A})$, we have

$$\sinh(A+B) = \frac{1}{2}\left[e^{A}e^{B} - e^{-A}e^{-B}\right] = \frac{1}{2}\left[(\cosh A+\sinh A)(\cosh B+\sinh B) - (\cosh A-\sinh A)(\cosh B-\sinh B)\right]$$

$$= \frac{1}{2}\left[\cosh A\cosh B + \cosh A\sinh B + \sinh A\cosh B + \sinh A\sinh B - \cosh A\cosh B + \cosh A\sinh B + \sinh A\cosh B - \sinh A\sinh B\right]$$

$$= \cosh A\sinh B + \sinh A\cosh B$$

& sim.,

$$\cosh(A+B) = \cosh A\cosh B + \sinh A\sinh B$$

then
$$\tanh(A+B) = \frac{\sinh(A+B)}{\cosh(A+B)} = \frac{\cosh A\sinh B + \sinh A\cosh B}{\cosh A\cosh B + \sinh A\sinh B}$$

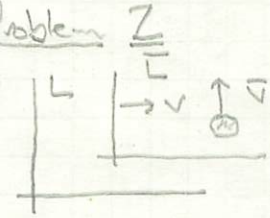
$$= \frac{\cosh A\sinh B + \sinh A\cosh B}{\cosh A\cosh B(1+\tanh A\tanh B)} = \frac{\tanh B + \tanh A}{1+\tanh A\tanh B}$$

Velocity addition reads: $V_h = \frac{V+\bar{V}}{1+V\bar{V}/c^2} \Rightarrow \tanh\chi = \frac{\tanh\eta + \tanh\psi}{1+\tanh\eta\tanh\psi}$

$$= \tanh(\eta+\psi)$$

so that $\chi = \eta + \psi$, rapidities add.

Problem 2



In $\bar{L}$: $\bar{v} = \frac{\Delta\bar{y}}{\Delta\bar{t}}$

In L: $v = \frac{\Delta y}{\Delta t} = \frac{\Delta\bar{y}\,c}{\gamma(c\Delta\bar{t} + v/c\,\Delta\bar{x})} = \frac{\bar{v}}{\gamma}$

so the velocity of the ball, in L, is: $\vec{v} = v\hat{x} + \bar{v}/\gamma\,\hat{y}$.

Problem 3

$x(t) = v_0 t$ – proper time is defined by: $\frac{dt}{d\tau} = \frac{1}{\sqrt{1-v^2/c^2}}$ w/ $v = \dot{x}(t)$,

so $t = \frac{\tau}{\sqrt{1-v_0^2/c^2}}$ & $x(\tau) = \frac{v_0\tau}{\sqrt{1-v_0^2/c^2}}$

Problem 4

$$\frac{d}{d\eta}\sinh\eta = \frac{d}{d\eta}\frac{1}{2}(e^{\eta}-e^{-\eta}) = \frac{1}{2}(e^{\eta}+e^{-\eta}) = \cosh\eta$$

We have $t = \alpha\sinh(\tau/\alpha)$ so $\frac{dt}{d\tau} = \cosh(\tau/\alpha) = \frac{1}{\sqrt{1-\dot{x}^2/c^2}}$

then $1-\dot{x}^2/c^2 = \frac{1}{\cosh^2(\tau/\alpha)} \Rightarrow \dot{x} = c\left[1-\frac{1}{\cosh^2(\tau/\alpha)}\right]^{1/2} = \left[\frac{\cosh^2(\tau/\alpha)-1}{\cosh^2(\tau/\alpha)}\right]^{1/2} c$

or

$$\dot{x} = \frac{c\sinh(\tau/\alpha)}{[1+\sinh^2(\tau/\alpha)]^{1/2}} = \frac{c\cdot t/\alpha}{[1+(t/\alpha)^2]^{1/2}}$$

$\to$

$$x(t) = \frac{c}{\alpha}\int_0^t \frac{\bar{t}}{[1+(\bar{t}/\alpha)^2]^{1/2}}\,d\bar{t} = \frac{c}{\alpha}\cdot\alpha^2\left[1+(t/\alpha)^2\right]^{1/2} = \alpha c\left[1+(t/\alpha)^2\right]^{1/2}$$

The force can be found from: $\frac{d}{dt}\left[\frac{m\dot{x}}{\sqrt{1-\dot{x}^2/c^2}}\right] = F$

$$1-\dot{x}^2/c^2 = 1 - \frac{t^2/\alpha^2}{1+(t/\alpha)^2} = \frac{1}{1+(t/\alpha)^2}$$

so $\frac{m\dot{x}}{\sqrt{1-\dot{x}^2/c^2}} = \frac{mc\,t/\alpha}{[1+(t/\alpha)^2]^{1/2}}\cdot\left[1+(t/\alpha)^2\right]^{1/2} = mc\,t/\alpha$

$\to$ $F = \frac{d}{dt}\left[\frac{m\dot{x}}{\sqrt{1-\dot{x}^2/c^2}}\right] = \frac{d}{dt}\left(\frac{mct}{\alpha}\right) = \frac{mc}{\alpha}$.

Problem 5

In one full cycle, $l = 2\pi R$..

$dx = -\omega R\sin(\omega t)\,dt \quad dy = \omega R\cos(\omega t)\,dt$, so

$ds^2 = -c^2dt^2 + dx^2 + dy^2 = -c^2dt^2\left[1-(\omega R/c)^2\right]$, $\to$

$$ds = ic\left[1-(\omega R/c)^2\right]^{1/2}dt \Rightarrow s = \int_0^{T=2\pi/\omega} ic\left[1-\left(\frac{\omega R}{c}\right)^2\right]^{1/2}dt$$

$$= ic\,2\pi/\omega\cdot\sqrt{1-\left(\frac{\omega R}{c}\right)^2}$$