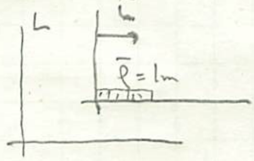
Problem 1

Problem Set 1



$l = {}^{12}/_{13}\ m\ ,\quad \bar{l} = 1m$

$l = \sqrt{1 - v^2/c^2}\,\bar{l} \Rightarrow (l/\bar{l})^2 = 1 - v^2/c^2 \Rightarrow v = c\sqrt{1-(l/\bar{l})^2}$

so here, $1 - (l/\bar{l})^2 = \frac{25}{169}$ so $v = \frac{5}{13}c$

Problem 2

In L, $d = v \cdot t \Rightarrow 5m = ({}^{12}/_{13})\,c \cdot t \Rightarrow t = \frac{5m}{({}^{12}/_{13})\,c} = \frac{65m}{12} \cdot \frac{1}{c}$

$\bar{t} = \gamma(t - v/c^2 d) = \gamma(1 - v^2/c^2)t = \sqrt{1 - v^2/c^2}\cdot t = \frac{5}{13}\cdot\frac{5m}{{}^{12}/_{13}c} = \frac{25m}{12}\cdot\frac{1}{c}$.

Problem 3

$\Delta t = \gamma \Delta\bar{t} \Rightarrow \Delta t = \frac{1}{\sqrt{1-(3/5)^2}}\cdot 2\text{years} = \frac{1}{4/5}\cdot 2\text{years} = 5/2\text{ years}$

Problem 4

a. $s^2 = -c^2(t_2 - t_1)^2 + (x_2 - x_1)^2 = \bar{s}^2 = -c^2(\bar{t}_2 - \bar{t}_1)^2 + (\bar{x}_2 - \bar{x}_1)^2$ in any $\bar{L}$ related to L by a Lorentz boost, so if $s^2 \le 0$, so, too, does $\bar{s}^2$ - the events are causally related in both frames.

b. B could cause C, D could cause C

Problem 5

$$\gamma_w^{-2} = 1 - \frac{1}{c^2}\left(\frac{v-u}{1-uv/c^2}\right)^2 = \frac{c^2(1 - 2uv/c^2 + u^2v^2/c^4) - v^2 + 2uv - u^2}{c^2(1-uv/c^2)^2}$$

$$= \frac{c^2(1 - v^2/c^2 - u^2/c^2 + u^2v^2/c^4)}{c^2(1-uv/c^2)^2}$$

$$= \frac{(1 - v^2/c^2)(1 - u^2/c^2)}{(1-uv/c^2)^2} = \gamma_v^{-2}\gamma_u^{-2}(1-uv/c^2)^{-2}$$

then: $\gamma_w^{-1} = \gamma_v^{-1}\gamma_u^{-1}(1-uv/c^2)^{-1} \Longrightarrow \gamma_w = \gamma_u\gamma_v(1-uv/c^2)$ ✓