Problem Set 9

Physics 322
Electrodynamics II

Due on Friday, April 5th, 2024

Problem 1

Find the intensity of Thomson scattering for a charge q (initially at rest at the origin) that responds to an incident plane electromagnetic wave with electric field of the form: $\mathbf{E} = E_0 \cos(k(z - ct))\hat{\mathbf{y}}$. Write your result in terms of spherical coordinates, the "electron radius," ℓ, and the incident intensity $I_0 = 1/2E_0^2/(\mu_0 c)$. Why doesn't the intensity depend on the frequency $\omega \equiv kc$?

Problem 2

From the plane wave solutions to the wave equation (working in one spatial dimension here)

$$w(t) = w_0 e^{i2\pi(ft-kx)} \qquad \text{with } f = kv, \tag{1}$$

we can make more general solutions by summing,

$$p(x,t) = \int_{-\infty}^{\infty} A(k) e^{i2\pi ft} e^{-i2\pi kx} dk. \tag{2}$$

where $A(k)$ is a function of the wavenumber (an inverse length) k.

a. Given $p(x, 0) = u(x)$ as the initial data, find the expression for $A(k)$ in terms of $u(x)$ (and functions related to it – it might help to review the definition of the Fourier transform from the notes for January 31st, 2024).

b. Suppose the frequency f is itself a function of the wavenumber, $f(k)$, so that

$$p(x,t) = \int_{-\infty}^{\infty} A(k) e^{i2\pi f(k)t} e^{-i2\pi kx} dk. \tag{3}$$

If $A(k)$ is sharply peaked about some k_0, then by Taylor expanding f about k_0 and using that expression in (3), show that

$$p(x,t) \approx u(x - f'(k_0)t) e^{i\phi t} \tag{4}$$

where ϕ is a constant. Up to a phase factor, then, $p(x,t)$ is interpretable as a right-traveling wave that moves with speed $f'(k_0)$ (the relevant group velocity here).

Problem 3

What is the energy required to build a spherical shell of radius R with total charge Q spread uniformly around its surface? How about a solid, uniformly charged sphere of radius R with that same charge Q? In each case, find the "classical charge radius."

Problem 4

Using the Thomson scattering setup, we can define the "scattering cross section" associated with electromagnetic radiation. For the incident wave: $\mathbf{E}_{\text{in}} = E_{\text{in}} \cos(k(z - ct))\hat{\mathbf{n}}$, the amount of power that goes through a disk of area $d\sigma$ (with the disk's surface normal pointing in the direction of the incident wave, $\hat{\mathbf{z}}$) is: $S_{\text{in}} d\sigma$. The radiation that comes out (Thomson scattering) goes through a patch of a sphere of radius r, with $d\mathbf{a} = r^2 \sin\theta d\theta d\phi \hat{\mathbf{r}}$. The angular piece of the area patch defines the solid angle, $d\Omega \equiv \sin\theta d\theta d\phi$. Equating the incoming power with the outgoing power:

$$S_{\text{in}} d\sigma = \mathbf{S}_{\text{out}} \cdot \left(r^2 d\Omega \hat{\mathbf{r}}\right) \tag{5}$$

allows us to define the "differential scattering cross section," the function $D(\theta) \equiv \frac{\left(r^2 \hat{\mathbf{r}} \cdot \mathbf{S}_{\text{out}}\right)}{S_{\text{in}}}$. Write out the expression for $D(\theta)$ for Thomson scattering using the time-averaged Poynting vectors (the intensities), and averaging over the incident polarization.

Problem 5

Griffiths 9.36 – transmission through multiple materials – feel free to use the matrix setup (or something similar to it) from problem 2 of Problem Set 8.

Problem 6

Griffiths 9.38 – Scaring fish.