Problem Set 8

Physics 322
Electrodynamics II

Due on Friday, March 29th, 2024

Problem 1

Griffiths 9.17 — Fresnel equations for electric field polarized perpendicular to the plane of incidence. Assume that $\beta = n_2/n_1$ when showing that there is no Brewster's angle.

Problem 2

Two different linear media meet up at $z = a$ as shown below. On the left, we have ϵ_1, μ_1, v_1 and n_1 (related in the usual way), and on the right, ϵ_2, μ_2, v_2 and n_2.

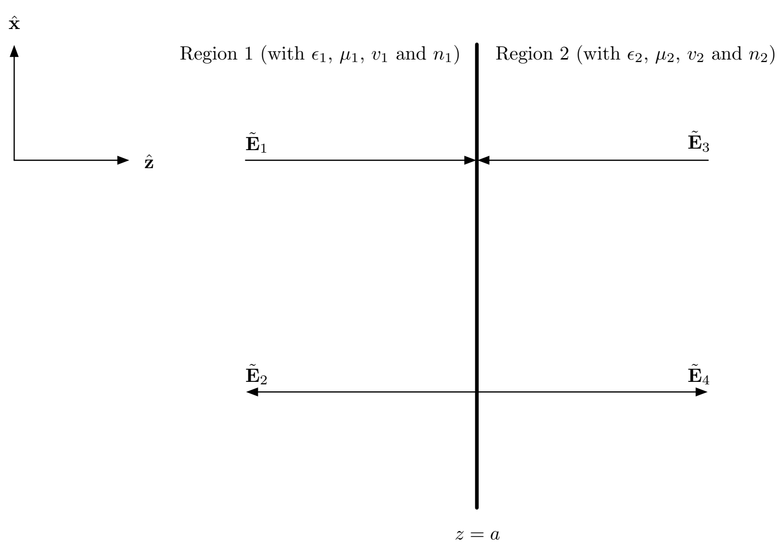

Figure 1: Setup for Problem 2

A plane wave electric field comes in from the left (at normal incidence), $\tilde{\mathbf{E}}_1 =$

$\tilde{E}_{01} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}}$, and another comes in from the right, $\tilde{\mathbf{E}}_3 = \tilde{E}_{03} e^{-i(k_2 z + \omega t)} \hat{\mathbf{x}}$. These each give rise to reflected/transmitted fields, which we can combine in each region – call them $\tilde{\mathbf{E}}_2 = \tilde{E}_{02} e^{-(ik_1 z + \omega t)} \hat{\mathbf{x}}$ on the left (a combination of the reflected wave from the left, and the transmitted wave from the right) and $\tilde{\mathbf{E}}_4 = \tilde{E}_{04} e^{i(k_2 z - \omega t)} \hat{\mathbf{x}}$ on the right (a combination of the transmitted wave from the left and the reflected wave from the right). Using the appropriate boundary conditions (for both the electric and associated magnetic fields), write the relation between $\tilde{E}_{01}$, $\tilde{E}_{03}$ (the "incoming" complex amplitudes) and $\tilde{E}_{02}$, $\tilde{E}_{04}$ (the outgoing ones) as a matrix equation of the form:

$$\mathbb{A} \begin{pmatrix} \tilde{E}_{01} \\ \tilde{E}_{03} \end{pmatrix} = \mathbb{B} \begin{pmatrix} \tilde{E}_{02} \\ \tilde{E}_{04} \end{pmatrix}, \tag{1}$$

i.e. find the matrices $\mathbb{A}$ and $\mathbb{B}$. Using a symbolic algebra package, compute $\mathbb{B}^{-1}\mathbb{A}$ that appears in

$$\begin{pmatrix} \tilde{E}_{02} \\ \tilde{E}_{04} \end{pmatrix} = \mathbb{B}^{-1}\mathbb{A} \begin{pmatrix} \tilde{E}_{01} \\ \tilde{E}_{03} \end{pmatrix}. \tag{2}$$

Check that you get the correct result for $\tilde{E}_{03} = 0$ and $a = 0$.

Problem 3

A particle of mass m travels from $z < 0$ into a region of non-zero potential energy U_0 with $z > 0$. If the "angle of incidence" is θ_i (associated with the incoming velocity vector $\mathbf{v}_i = v_1 \sin\theta_i \hat{\mathbf{x}} + v_1 \cos\theta_i \hat{\mathbf{z}}$), what is the transmitted angle θ_t associated with the "outgoing" velocity vector $\mathbf{v}_t$ with speed v_2? Write your relation in the form of Snell's law, relating $\sin\theta_i$, $\sin\theta_t$, v_1 and v_2. The setup is shown in Figure 2. What is the relation between v_1 and v_2 in terms of U_0?

Problem 4

Suppose you have a hard sphere of radius R centered at the origin. You throw a ping-pong (or other light ball) at the sphere a distance b above the sphere's center (as shown in Figure 3) – at what angle, θ, does the ball bounce off the sphere? Hint: what is this problem doing in an E&M II problem set?

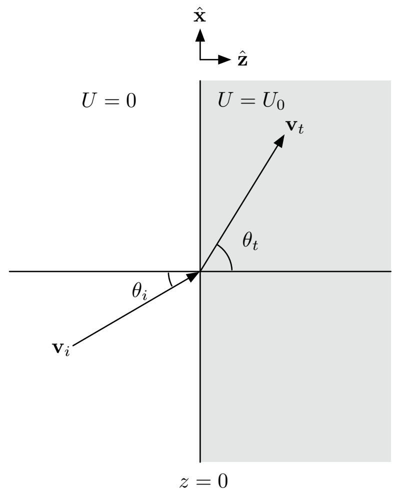

Figure 2: Setup for Problem 3

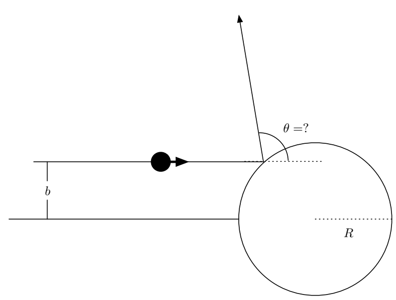

Figure 3: Setup for Problem 4

Presentation Problems

Here are some presentation problems (see syllabus for a description of the writeup/in-class presentation). Everyone should solve these problems – if you choose to present one, either in-class or in written form, it will be due on Friday, March 29th (I'll ask for volunteers for in-class presentations on Monday, March 25th).

Problem 1*

Suppose you have an electric charge q, and a magnetic charge (a monopole)

q_m with field

$$\mathbf{B} = \frac{\mu_0 q_m}{4\pi \mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} \, . \tag{3}$$

The two charges are separated by a distance d. Find the angular momentum in the fields of these charges. What would happen if you "quantized" this angular momentum, $L = n\hbar$ for integer n?

Problem 2*

Griffiths 9.39 – evanescent waves.