Problem Set 5

Physics 322
Electrodynamics II

Due on Friday, February 23rd, 2024

Problem 1

a. A charge q oscillates up and down along the $\hat{\mathbf{y}}$ axis, with $\mathbf{w}(t) = d\cos(\omega t)\hat{\mathbf{y}}$. We are interested in the field at location $\mathbf{r} = (d/100)\hat{\mathbf{x}} - d/10\hat{\mathbf{y}}$ and time $t = 0$. If we set $\omega = 10000$ Hz, $d = 20000$ m, what is the proper time associated with our field point and time? You can use root-finding on the function:

$$F(p) = c(t - p) - \sqrt{(\mathbf{r} - \mathbf{w}(p)) \cdot (\mathbf{r} - \mathbf{w}(p))} \tag{1}$$

in `Mathematica` via `FindRoot`. Where is the particle at t_r? Where is the particle at time 0?

b. Find t_r for the same motion and field point as in part a., but with $\omega = 30000$ Hz, $d = 40000$ m. (try plotting $F(p)$ before finding the root ...what is going on here?).

Problem 2

Griffiths 11.12 – electron falling under the influence of gravity.

Problem 3

A particle of charge q moves along a trajectory given by: $\mathbf{w}(t) = f\,\cos(\omega\,t)\,\hat{\mathbf{x}} + g\,\sin(\omega\,t)\,\hat{\mathbf{y}}$ (for constants f, g and ω) – sketch the trajectory of the particle (take $f > g$ for the sketch). What is the total power (time-averaged) radiated? Which radiates more, a particle moving in a circular trajectory of radius f, or a particle moving along a line of length $2\,f$ (each traversed with constant angular frequency ω)? What is the ratio of the power (time-averaged) radiated for the circle vs. the line trajectories?

Problem 4

For a configuration of charge that has dipole moment $\mathbf{p}(t) = p_0(\cos(\omega t)\hat{\mathbf{x}} + \sin(\omega t)\hat{\mathbf{y}})$, find $\mathbf{E}^{\text{rad}}$ and $\mathbf{B}^{\text{rad}}$ and the associated Poynting vector (for a field point at $\mathbf{r}$ with $r \gg p_0/q$ where q is the total charge of the configuration). Express all your answers in term of spherical coordinates and the spherical basis vectors.

Problem 5

A particle in "hyperbolic" motion travels along the z-axis with $\mathbf{w}(t) = \sqrt{b^2 + (ct)^2}\hat{\mathbf{z}}$.

What is the retarded time for a field point $\mathbf{r} = x\hat{\mathbf{x}} + z\hat{\mathbf{z}}$, and $t = 0$? What happens to the retarded time when $z \to 0$?

Problem 6

A point charge sits at the origin of $\bar{L}$ that moves through L with speed v along a shared x axis. Write down the charge density in $\bar{L}$ and L separately, using the appropriate point charge density in each frame. For your charge density in $\bar{L}$, express $\bar{x}$, $\bar{y}$ and $\bar{z}$ in terms of the coordinates in L and use properties of the delta function to show that $\rho = \gamma\bar{\rho}$. Interestingly, then, length contraction occurs even for point charges ...

Presentation Problems

Here are some presentation problems (see syllabus for a description of the writeup/in-class presentation). Everyone should solve these problems – if you choose to present one, either in-class or in written form, it will be due on Friday, February 23rd (I'll ask for volunteers for in-class presentations on Monday, February 19th).

Problem 1*

Construct a function in `Mathematica` (or other program) that will evaluate the full electric field $\mathbf{E}$ from Griffiths (10.72) and $\mathbf{B}$ (10.73) at field points $\mathbf{r} = y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and time t. Try it out (and come up with some mechanism for display) on some of the motion that we have been thinking about radiatively in class.

Problem 2*

Griffiths 11.14 – lifetime of hydrogen (classical).