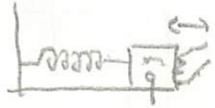
## Last Time

We developed the radiation reaction force:

$$\vec{F} = \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}$$

a mechanical accounting scheme that matches the Larmor power loss.

For a driven mass on a spring




we get a force that looks like damping.

For a free particle, we get

$$m\ddot{x}(t) = m\tau\dddot{x}(t)$$

$$\tau = \frac{\mu_0 q^2}{6\pi m c}$$

w/

$$a(t) = a_0 e^{t/\tau}$$

as the acceleration ...

## Landau-Lifschitz Force.

There's a reformulation of the radiation reaction force that sidesteps some of the problems.

Working in D=1, & using "s" as the temporal parameter — suppose you apply an external force F:

$$m\ddot{x}(s) = F(s) + m\tau\dddot{x}(s)$$

rad. reaction force

or

$$m\big(\underbrace{\ddot{x}(s) - \tau\dddot{x}(s)}_{\approx\,\ddot{x}(s-\tau)}\big) = F(s)$$

$$m\ddot{x}(s-\tau) = F(s) \quad \text{let } t \equiv s-\tau,$$

$$m\ddot{x}(t) = F(t+\tau) \approx F(t) + \underbrace{\tau\dot{F}(t)}_{\text{L-L force.}}$$

In this form, if there is no external force, there is no radiation-reaction force.

For a free particle: $F=0$ & $m\ddot{x}=0$ ✓

example — take a spring:


$F = -kx$

then $\dot{F} = -k\dot{x}$, &

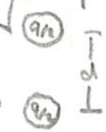
$$m\ddot{x} = F + \tau\dot{F} = -kx - (k\tau)\dot{x}$$

a damping force ✓

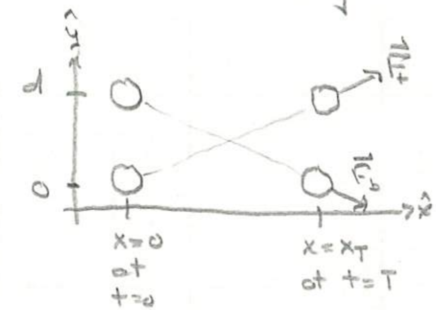
## Self Force

idea — model a charge q as a pair of charges q/2:

$q = \lim_{d\to 0}$




Consider the following setup:




At t=0, the pair is at x=0.

At t=T, the pair is at $x_T$.

let t=0 be the retarded time for the location of the top charge:

$$c(T-0) = \sqrt{x_T^2+d^2} \qquad (*)$$

then field information from the bottom charge at t=0 reaches the top charge at T, & exerts a force on the top charge.
(& sim. for the force exerted on the bottom charge at T from the top charge at 0)

The electric field of a charge moving along $\vec{w}(t)$ is

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{r}{(\vec{r}\cdot\vec{u})^3}\left[(c^2-v^2)\vec{u} + \vec{r}\times(\vec{u}\times\vec{a})\right]$$

we can chose a trajectory.

let the $\hat{x}$ location of the dumbbell be:

$x(t) = \frac{1}{6} j_0 t^3$, then at the retarded time $t=0$:

$x(0) = 0$, $\dot{x}(0) = \frac{1}{2} j_0 t^2 \big|_{t=0} = 0$, $\ddot{x}(0) = j_0 t \big|_{t=0} = 0$

working out the pieces of $\vec{E}$:

$$\vec{\mathscr{r}} = \vec{r} - \vec{w}(t_r) = (x_T - x(0))\,\hat{x} + (d - 0)\,\hat{y}$$

for lower charge

$$= x_T \hat{x} + d\hat{y}, \text{ w/ } \mathscr{r} = \sqrt{x_T^2 + d^2}$$

$$\vec{u} = c\hat{\mathscr{r}} - \underbrace{\vec{v}}_{(=0 \text{ at } t=0)} = \frac{c(x_T\hat{x} + d\hat{y})}{\sqrt{x_T^2+d^2}}$$

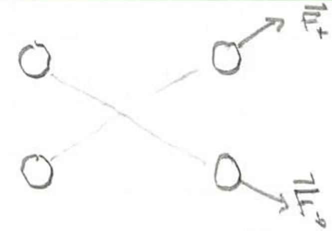
w/ $\vec{u}\cdot\vec{\mathscr{r}} = c\sqrt{x_T^2+d^2}$

$$\vec{E} = \frac{q/2}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{u}\cdot\vec{\mathscr{r}})^3}\left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\underbrace{\vec{a}}_{(0 \text{ at } t=0)})\right]$$

$$= \frac{q/2}{4\pi\epsilon_0}\frac{\mathscr{r}\hat{\mathscr{r}}}{(x_T^2+d^2)^{3/2}} = \frac{q/2}{4\pi\epsilon_0}\frac{(x_T\hat{x} + d\hat{y})}{(x_T^2+d^2)^{3/2}}.$$

& from (*), we have: $(x_T^2 + d^2)^{3/2} = c^3T^3$

so $\vec{E} = \frac{q/2}{4\pi\epsilon_0}\frac{(x_T\hat{x} + d\hat{y})}{c^3T^3}$




we've been focused on the top charge, w/

$$\vec{F}_+ = q/2\,\vec{E} = \frac{(q/2)^2}{4\pi\epsilon_0}\frac{(x_T\hat{x} + d\hat{y})}{c^3T^3}$$

but the force on the bottom charge is easy to get from $\vec{F}_+$:

$$\vec{F}_b = \frac{(q/2)^2}{4\pi\epsilon_0}\frac{(x_T\hat{x} - d\hat{y})}{c^3T^3}$$

the net force on the dumbbell is:

$$\vec{F} = \vec{F}_+ + \vec{F}_b = \frac{2(q/2)^2}{4\pi\epsilon_0}\frac{x_T\hat{x}}{c^3T^3} \quad \& \quad x_T = \tfrac{1}{6} j_0 T^3$$

so $$\vec{F} = \frac{2(q/2)^2}{4\pi\underbrace{\epsilon_0 c^2}_{=1/\mu_0}}\cdot\frac{\frac{1}{6}j_0}{c}\hat{x} = \frac{\mu_0 q^2 j_0}{48\pi c}\hat{x}$$

& $j_0 = \dddot{x}(0) = \dot{a}(t_r) = \dot{a}(T)$ here, a peculiarity of our model.

$$\vec{F} = \frac{\mu_0 q^2 \dot{\vec{a}}(t)}{48\pi c}, \text{ \& take } \lim d\to 0 \text{ to get } (9)$$

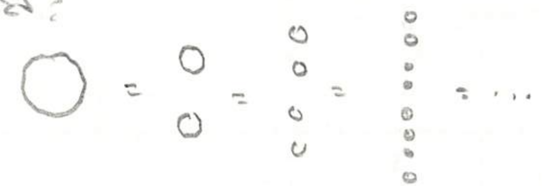
The radiation reaction force is (up to constants for now) the force of a point charge on itself ...

We should have $F = \frac{\mu_0 q^2 \dot{a}}{12\pi c}$ at this stage (see textbook).

That's still not correct.

The issue is that we took $(q) = \begin{matrix} (q/2) \\ (q/2) \end{matrix}$ $\Big\updownarrow d$ as $d \to 0$.

But what about those $(q/2)$ charges? Can't they be broken up into 2 $q/4$ charges? etc.?



let the contribution we just calculated be $F_1$ then

$$F = \underset{(q)}{F_1} + \underset{(q/2)}{F_2} \qquad (*)$$

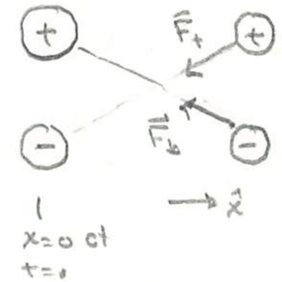
in $\hat{x}$ direction w/ sign giving $+\hat{x}$ or $-\hat{x}$

we want to know $F_2$.

= Consider another configuration – a neutral particle that we model as a pair of equal + opposite charges separated by d:

$$(0c) = \lim_{d \to 0} \begin{matrix} (q/2) \\ \\ (-q/2) \end{matrix}$$

trouble:




$\vec{F} = -F_1 \hat{x}$

same thing we just calculated, but points backwards.

the net force on the neutral particle should be 0

$F = -F_1 + F_2$ ← this $F_2$ refers to $(+) = \begin{matrix} (+q/2) \\ (+q/2) \end{matrix}$ same sign

& $(-) = \begin{matrix} (-q/2) \\ (-q/2) \end{matrix}$ same sign

i.e. in both of these cases, we have our original setup, w/ forces pointing to the right $(+\hat{x})$

To get $F = -F_1 + F_2 = 0 \Rightarrow F_2 = F_1$

and using this information back in $(*)$ gives

$$F = F_1 + F_1 = 2F_1$$

or $$\vec{F} = 2\left(\frac{\mu_0 q^2 \dot{a}}{12\pi c}\right) = \frac{\mu_0 q^2 \dot{a}}{6\pi c} \quad ✓$$