

Problem 1 (1.2)

Problem Set 1

No-counterexample:  $\vec{A} = \hat{x}$ ,  $\vec{B} = \hat{x}$ ,  $\vec{C} = \hat{y}$ , then

$$(\vec{A} \times \vec{B}) \times \vec{C} = (\hat{x} \times \hat{x}) \times \hat{y} = 0 \quad \text{but}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \hat{x} \times (\hat{x} \times \hat{y}) = \hat{x} \times \hat{z} = -\hat{y} \neq 0.$$

Problem 2 (1.4)

Let  $\vec{A} = 3\hat{z} - \hat{x}$  &  $\vec{B} = 2\hat{y} - \hat{x}$ , then  $\hat{n} \parallel \vec{B} \times \vec{A}$

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\hat{x} + 3\hat{y} + 2\hat{z} = \vec{n}$$

$$\therefore n = (36 + 9 + 4)^{1/2} = 7 \quad \therefore \hat{n} = \vec{n}/n = 1/7(6\hat{x} + 3\hat{y} + 2\hat{z}).$$

Problem 3

$$\vec{B} = B^x \hat{x} + B^y \hat{y}, \quad \vec{C} = C^x \hat{x} + C^y \hat{y}, \quad \vec{A} = A^x \hat{x} + A^y \hat{y} + A^z \hat{z}$$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B^x & B^y & 0 \\ C^x & C^y & 0 \end{vmatrix} = \hat{z}(B^x C^y - B^y C^x) = \vec{D}$$

$$\begin{aligned} \therefore \vec{A} \times (\vec{B} \times \vec{C}) &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A^x & A^y & A^z \\ 0 & 0 & D^z \end{vmatrix} = \hat{x}(A^y D^z) - \hat{y} A^x D^z \\ &= (B^x C^y - B^y C^x) [A^y \hat{x} - A^x \hat{y}] \end{aligned}$$

$$\begin{aligned} \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) &= (A^x C^x + A^y C^y)(B^x \hat{x} + B^y \hat{y}) - (A^x B^x + A^y B^y)(C^x \hat{x} + C^y \hat{y}) \\ &= (B^x C^y - B^y C^x) A^y \hat{x} + (B^y C^x - B^x C^y) A^x \hat{y} \\ &= (B^x C^y - B^y C^x) (A^y \hat{x} - A^x \hat{y}) = \vec{A} \times (\vec{B} \times \vec{C}) \end{aligned}$$

Problem 4

$$\vec{A} = 3\hat{x} - 2\hat{y} + \hat{z}, \quad \vec{B} = 2\hat{x} + \hat{y} - 2\hat{z} \quad \text{w/ } A = \sqrt{14}, \quad B = 3$$

$$\vec{A} \cdot \vec{B} = 6 - 2 - 2 = 2 = 3\sqrt{14} \cos \theta \Rightarrow \cos \theta = \frac{2}{3\sqrt{14}} \Rightarrow \theta = \cos^{-1} \left( \frac{2}{3\sqrt{14}} \right) = 1.39$$

$$\text{or } \theta = 1.39 \cdot \frac{360^\circ}{2\pi} \approx 79.6^\circ$$