Assignment 5

Physics 321
Electrodynamics I

Due on Friday, September 20th, 2024

Class date: September 13th, 2024.
Reading: pp. 43–50, Dirac delta function, pp. 57-60, charges and the electric field.

Problem 1

You showed that the curl of the vector field

$$\mathbf{A} = \frac{-y\hat{\mathbf{x}} + x\hat{\mathbf{y}}}{x^2 + y^2} \tag{1}$$

was zero in Problem Set 2. That was surprising since $\mathbf{A}$ clearly "curls" around the origin. Use Stokes' theorem in cylindrical coordinates to express the curl of $\mathbf{A}$ in terms of the two-dimensional delta function: $\delta^2(x, y) = \delta(x)\delta(y)$.

Problem 2

Using the symmetric form of the Dirac delta distribution: $\delta(-x) = \delta(x)$, what is

$$\int_0^\infty \delta(x)dx = ? \tag{2}$$

Problem 3

Griffiths 1.46 – Derivatives of the Dirac delta distribution and the step function. Note that to show that an expression behaves like a delta function, you can use the "definition" in (1.92), i.e. $g(x)$ "is" a Dirac delta distribution if

$$\int_{-\infty}^\infty f(x)g(x)dx = f(0) \tag{3}$$

for any $f(x)$. For the step function in part b., what must its value at zero be given your result from Problem 2?

Presentation Problems

Here are two presentation problems – if you are interested in writing one of them up, the writeup will be due on Friday, September 27th (send a pdf to me via e-mail). Instructions for the format of the writeup can be found in the course syllabus, and an example writeup is on the course website.

Problem 1*

Suppose the fundamental force between two charges q and Q looked like:

$$\mathbf{F} = \frac{kqQ}{r}\hat{\mathbf{r}} \tag{4}$$

where k is a constant. The difference here, with respect to the Coulomb force, is that the magnitude only goes like $1/r$. Assuming superposition holds, what is the general integral solution for the electric field at location $\mathbf{r}$ given charge density ρ? What is the divergence and curl of this electric field?

Problem 2*

We saw that $\nabla \cdot \mathbf{E}(\mathbf{r}) = \rho(\mathbf{r})/\epsilon_0$, and we will typically use this equation to find the electric field, $\mathbf{E}$, given a charge density ρ. But the equation can be "turned around" — given $\mathbf{E}$, find the charge distribution that generated it. Give it a shot with the following electric fields:

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} \quad \mathbf{E} = \alpha r\hat{\mathbf{r}} \tag{5}$$

where r is the spherical r and $\hat{\mathbf{r}}$ is the spherical basis vector associated with r, α is a constant. Do the charge distributions here make sense?

Next try the following — again, find ρ from $\mathbf{E}$, all coordinates and basis vectors are Cartesian:

$$\mathbf{E} = \alpha z\hat{\mathbf{z}} \quad \mathbf{E} = \alpha y\hat{\mathbf{y}} \quad \mathbf{E} = \beta\hat{\mathbf{x}} \quad \mathbf{E} = \beta\hat{\mathbf{y}}. \tag{6}$$

In each case, α and β are constants. Are the source distributions that generate these electric fields sensible?