Assignment 3

Physics 321
Electrodynamics I

Due on Friday, September 13th, 2024

Class date: September 9th, 2024.
Reading: pp. 24–37, review of vector calculus integrals.

Problem 4

For the vector function $\mathbf{v} = (2xyz + yz^3)\hat{\mathbf{x}} + (x^2z + xz^3)\hat{\mathbf{y}} + (x^2y + 3xyz^2)\hat{\mathbf{z}}$, evaluate the line integral $I = \int_P \mathbf{v} \cdot d\boldsymbol{\ell}$ connecting the point $\mathbf{a} = \{0, 0, 0\}$ to $\mathbf{b} = \{2, 1, 2\}$ along the curve $\mathbf{w}(s) = 2s\hat{\mathbf{x}} + s^2\hat{\mathbf{y}} + 2s\hat{\mathbf{z}}$ (where the vector $\mathbf{w}(s)$ points from the origin to a point along the curve, parametrized by s) for $s = 0 \to 1$.

Problem 5

In the previous problem, $\mathbf{v} = \nabla T$ for some function T. Find that function, and check the "fundamental theorem for gradients" by comparing your result from the path integral in Problem 1 with $T(\mathbf{b}) - T(\mathbf{a})$ (à la (1.55)).

Problem 6

Griffiths 1.33 – Testing the divergence theorem.

Problem 7

Griffiths 1.61 parts a-e – Various alternate forms of the fundamental theorem of Calculus in the vector setting. For the ones involving a constant vector $\mathbf{c}$, try to put your expressions in the form: $\mathbf{c} \cdot \left(\int\right) = \mathbf{c} \cdot \left(\int\right)$, i.e. pull out a common factor of $\mathbf{c}$ dotted (or crossed) into the expressions appearing on the left and right (since $\mathbf{c}$ is a constant, it can come out of whatever type of integral you may have) — then you can eliminate $\mathbf{c}$ from both sides provided you use the "magic words": Since $\mathbf{c}$ is an arbitrary constant vector, take $\mathbf{c} = 1\hat{\mathbf{x}}$, so that the x-components of the integrals are equal, then take $\mathbf{c} = 1\hat{\mathbf{y}}$ to establish that the y-components of the integrals are equal, and finally, set $\mathbf{c} = \hat{\mathbf{z}}$, the z-components of the integrals are equal, so the integrals must be equal, and you can erase the $\mathbf{c}\cdot$ from both sides. The same sort of argument works when you get $\mathbf{c} \times \left(\int\right) = \mathbf{c} \times \left(\int\right)$.