At the End of the Semester

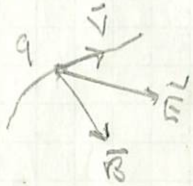
Given ρ & $\vec{J}$:

$\nabla \cdot \vec{E} = \rho/\epsilon_0 \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

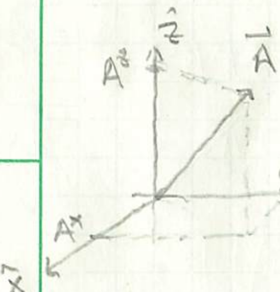
Maxwell's eqn.s tell us how to find $\vec{E}$ & $\vec{B}$

The Lorentz force tells us how charges respond to $\vec{E}$ & $\vec{B}$



$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Vectors

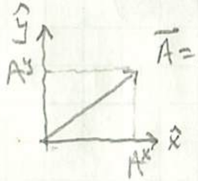


$\vec{A} = A^x \hat{x} + A^y \hat{y} + A^z \hat{z}$

components

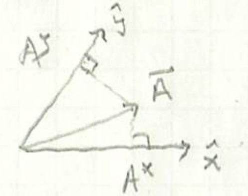
basis

or, "at the board"

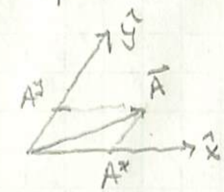


$\vec{A} = A^x \hat{x} + A^y \hat{y}$

Aside - there are "projection options"



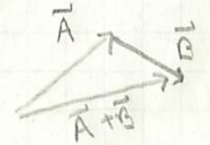
vs.



"perpendicular" projection "parallel" projection

Vector Operations

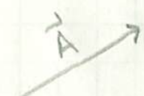
Addition: $\vec{A} = A^x \hat{x} + A^y \hat{y} \qquad \vec{B} = B^x \hat{x} + B^y \hat{y}$



$\vec{A} + \vec{B} = (A^x + B^x)\hat{x} + (A^y + B^y)\hat{y}$

Multiplication

1. by a scalar $\alpha \in \mathbb{R}$



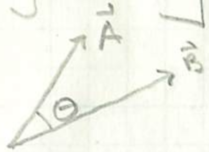
$\alpha\vec{A} = \alpha A^x \hat{x} + \alpha A^y \hat{y} \qquad \alpha\vec{A} \parallel \vec{A}$

2. dot product $\vec{A} \cdot \vec{B} = A^x B^x + A^y B^y$

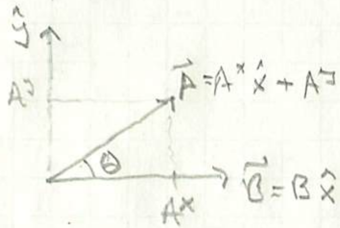
note that $\vec{A} \cdot \vec{A} = (A^x)^2 + (A^y)^2$, & the "length" of a vector is:

$$A \equiv \sqrt{\vec{A} \cdot \vec{A}} = \left[(A^x)^2 + (A^y)^2\right]^{1/2}$$

graphically:



rotate head

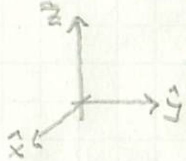


$\vec A\cdot\vec B = A^x B$ w/ $A^x = A\cos\theta$

so

$$\vec A\cdot\vec B = AB\cos\theta.$$

3. cross product – defined by action on axes:



$\hat x\times\hat x = 0 \quad \hat x\times\hat y = \hat z \quad \hat x\times\hat z = -\hat y$

$\hat y\times\hat x = -\hat z \quad \hat y\times\hat y = 0 \quad \hat y\times\hat z = \hat x$

$\hat z\times\hat x = \hat y \quad \hat z\times\hat y = -\hat x \quad \hat z\times\hat z = 0$

For $\vec A = A^x\hat x + A^y\hat y + A^z\hat z$, $\vec B = B^x\hat x + B^y\hat y + B^z\hat z$

$$\vec A\times\vec B = (A^x\hat x + A^y\hat y + A^z\hat z)\times(B^x\hat x + B^y\hat y + B^z\hat z)$$

$$= A^xB^x(\hat x\times\hat x) + A^xB^y(\hat x\times\hat y) + A^xB^z(\hat x\times\hat z)$$

$$+ A^yB^x(\hat y\times\hat x) + A^yB^y(\hat y\times\hat y) + A^yB^z(\hat y\times\hat z)$$

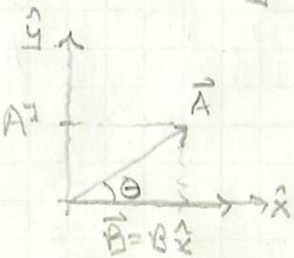
$$+ A^zB^x(\hat z\times\hat x) + A^zB^y(\hat z\times\hat y) + A^zB^z(\hat z\times\hat z)$$

$$= (A^xB^y - A^yB^x)\hat z - (A^xB^z - A^zB^x)\hat y$$

$$+ (A^yB^z - A^zB^y)\hat x$$

shorthand: $\vec A\times\vec B = \begin{vmatrix} \hat x & \hat y & \hat z \\ A^x & A^y & A^z \\ B^x & B^y & B^z \end{vmatrix} \leftarrow$ det

graphically – post-head rotation:



$\vec A = A^x\hat x + A^y\hat y$, $\vec B = B\hat x$

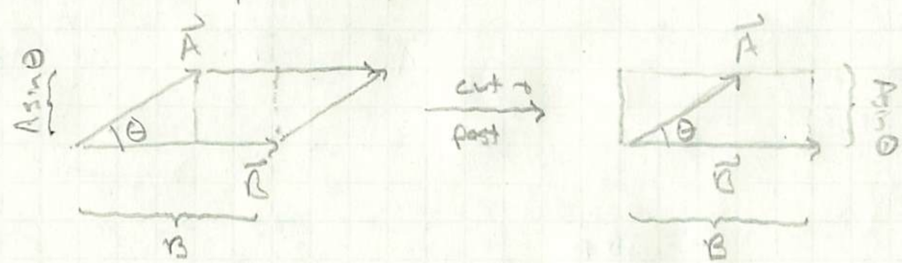
$\vec A\times\vec B = -A^yB\hat z$

direction mnemonic – right hand rule

magnitude is $\|\vec A\times\vec B\| = A^yB$ w/ $A^y = A\sin\theta$

$= AB\sin\theta.$

area interpretation:



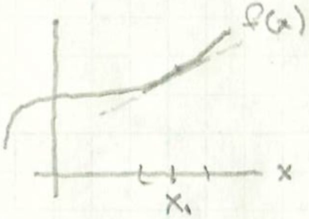
area is $AB\sin\theta = \|\vec A\times\vec B\|$.

identities: $\vec A\times\vec B = -\vec B\times\vec A$ $\quad \vec A\cdot(\vec A\times\vec B) = 0 \leftarrow$?why

$\vec A\times(\vec B\times\vec C) = \vec B(\vec A\cdot\vec C) - \vec C(\vec A\cdot\vec B)$

unit vectors: given $\vec A$, $\hat A = \vec A/A$ is $\parallel \vec A$ + has $\hat A\cdot\hat A = \frac{\vec A\cdot\vec A}{A^2} = 1$

Derivatives



$$f'(x_0) = \left.\frac{df(x)}{dx}\right|_{x=x_0}$$

gives slope of tangent line at x_0.

$f'(x_0) > 0$ as above $\Rightarrow$ function increases to the right

? Given $f(x)$ & x_0, pick dx $\ni$

$$f(x_0 + dx) \geq f(x_0)$$

how should we choose dx ?

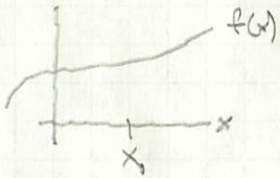
Taylor expansion:

$$f(x_0 + dx) \approx f(x_0) + dx\, f'(x_0) + \cdots$$

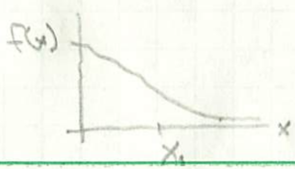
we want $dx\, f'(x_0) \geq 0$, so pick

$$dx \propto f'(x_0)$$

example:



$f'(x_0) > 0$, pick $dx > 0$



$f'(x_0) < 0$ pick $dx < 0$.