A First Course in Modular Forms: Corrections to the Third Printing

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(*The corrections here are also corrections to the earlier printings, but the third printing's pagination has changed a bit. In case of problems locating a correction here in an earlier printing, please email* `jerry@reed.edu`.)

Chapter 1

- Page 6, line (−9): Change "ring" to "Abelian group".

Chapter 2

- Page 55, line 2: Change "$\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$" to "$\gamma = \left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$".
- Page 55, line (−6): Change "proving (1)" to "proving (c)".
- Page 56, exercise 2.3.2: Change "If" to "If the nontrivial transformation".
- Page 61, line 14: Change "width" to "period".

Chapter 3

- Page 66, line 20: Change "equal genus" to "equal genus $g \geq 1$".
- Page 69, lines 4–5 (and the relevant bibliography item): Helena Verrill's fundamental domain drawer is at
  `http://www.math.lsu.edu/ verrill/fundomain/`
  on April 8, 2008.
- Page 70, line (−14): Change "of order 4" to "with $j' \neq j$".
- Page 70, line (−6): Change "$-6, \dots, 7$" to "$-6, \dots, 6, \infty$".
- Page 70, line (−5): Change "of order 3 or 6" to "with $j' \neq j$".
- Page 70, line (−1): Change "with with" to "with".
- Page 72: The quantity denoted h in lines 4–7 should be given a different name such as $\hbar$, as it is not necessarily the h or the h' in the discussion on page 74. The sentence "Thus f has period $\hbar$." on lines 4–5 is correct, but $\hbar$ need not be the smallest period of f.
- Page 74, line (−9): Change "$q_{h'} = e^{2\pi i/h'}$" to "$q_{h'} = e^{2\pi i\tau/h'}$".
- Page 74, line (−8): Delete "$q_{h'} = e^{2\pi i\tau/h'}$ and".

- Pages 74–75: The discussion in the "Defining. . . " paragraph on page 74 has an error: the period is $2h$ in the third case independently of k, even though $f(\tau+h) = f(\tau)$ for k even. That is, in the first two cases we have $h' = 2\hbar = 2h$ but in the third case we have $h' = \hbar = 2h$. On page 75, remove "and k is odd" from (3.3), and change the text immediately following, from "This can be half-integral in the exceptional case, when $\pi(s)$ or s itself is called an *irregular cusp* of Γ. For example, when k is odd $1/2$ is an irregular cusp. . . " to "This is half-integral if $(\alpha^{-1}\Gamma\alpha)_\infty = \langle -\left[\begin{smallmatrix} 1 & h \\ 0 & 1 \end{smallmatrix}\right]\rangle$ (when $\pi(s)$ or s itself is called an *irregular cusp* of Γ) and k is odd. For example, $1/2$ is an irregular cusp. . . ".
- Page 81, paragaph beginning "On the other hand...": Replace the discussion leading up to (3.7) with "On the other hand, if U_j contains a cusp s_j then δ_j takes s_j to ∞ and the function $(f[\alpha]_{2n})(z)$ takes the form $g_j(q_h)$ where h is the width of s and $q_h = e^{2\pi iz/h}$; here g_j is meromorphic in q_h at 0 if the cusp is regular and g_j is meromorphic in $q_h^{1/2} = e^{\pi iz/h}$ at 0 if the cusp is irregular, but we think of g_j as a series in powers of q_h (half-integral powers in the irregular case) so that the order is the index of the leading coefficient. The relevant local differential is now".
- Page 81, line (−7): Change "$\Omega^{k/2}(\mathcal{H})$" to "$\Omega^{\otimes k/2}(\mathcal{H})$".
- Page 95, line 4: Change "γ" to "γ_J".
- Page 95, line 15: Change "$\gamma =$" to $\gamma = \det m\cdot$".
- Section 3.7: A more transparent approach comes from the moduli space point of view, identifying $Y_0(N)$ and $\mathrm{S}_0(N)$ as in Theorem 1.5.1. For $N = 1$, elliptic points of $Y_0(N)$ correspond to elliptic curves $\mathbf{C}/\Lambda_\tau$ with automorphisms other than multiplication by ± 1. Since only two imaginary quadratic orders have more than two units, and they are both PID's, there are two elliptic points: one of order 2 corresponding to $\Lambda_\tau = i\mathbf{Z} \oplus \mathbf{Z}$, and one of order 3 corresponding to $\Lambda_\tau = \mu_3\mathbf{Z} \oplus \mathbf{Z}$. For $\Gamma_0(N)$, reason likewise. To find elliptic points of order 3, for example, look at the order N cyclic subgroups of the lattice $\mu_3 N\mathbf{Z} \oplus N\mathbf{Z}$, and count how many of them are invariant under multiplication by μ_3. These are precisely the subgroups generated by $m\mu_3 + 1$ where $m^2 - m + 1 \equiv 0 \pmod N$. Thus the number of elliptic points of order 3 is the number of solutions of the congruence $m^2 = m + 1 \equiv 0 \pmod N$.
- Page 106, fifth line of section 3.9: Change "$X_1(N)$" to "$X(1)$".

Chapter 4

- Page 123, line 9: Change "$a_{n^{-1}}(k)$" to "$a_n(k)$".
- Page 123, exercise 4.4.1(c): Change "$\mathrm{Re}(s) > 1$" to "$\mathrm{Re}(s) > 0$".
- Page 127, line 13: Change "$e' \equiv (e + c'b_\gamma)d_\gamma \pmod u$" to "$e' \equiv (e + c'b_\gamma)d_\gamma - q \pmod u$, where $q = (d' - dd_\gamma)/v$".
- Page 136, line 11: Change "negated" to "preserved".
- Page 136, line 12: Change "$t > 0$" to "$t < 0$".
- Page 136, line 13: Change "$\int_{t=-\infty}^{0}$" to "$\int_{t=0}^{-\infty}$".

- Page 137, line 9: Change "(n)" to "(k)".
- Page 140, lines 8–10: Delete the paragraph starting "Note that…".
- Page 142, lines 20–21: Remove exercise 4.8.6.
- Page 155, line 5: Change "$g(\bar{\varphi})/v$" to "$\varphi(-1)g(\bar{\varphi})/v$".
- Page 155, line 11: Change "$(-1)^k$" to "$\psi(-1)$".
- Page 155, line 19: Remove "$\varphi(-1)$".

Chapter 5

- Page 174, diagram (5.8): Change "$\mathcal{D}$" to "Div" four times.
- Page 186, line 4: Change "$\beta_j' = \det(\beta)\beta^{-1}$" to "$\beta_j' = \det(\beta_j)\beta_j^{-1}$".
- Page 192, line 8: Delete "$\pi_{d_1 d_2} =$".
- Page 204, second line of section 5.10: Change "n^s" to "n^{-s}".
- Page 204, line (-3): Change "idempotent" to "an involution".
- Page 205, line 4: Delete "under the Hecke algebra".
- Page 206, line 3: Change "idempotent" to "an involution".
- Section 5.11: The calculation of orthogonality is formally correct, but the absolute convergence of the double integral is not supported correctly by the text.
- Page 209, line 1: Change $f(\alpha(\tau'))$ to $(f[\alpha]_k)(\tau')$.
- Page 209, lines 2–3: Delete "if $\mathrm{Re}(k+2s) > 0$".

Chapter 6

- Page 212, lines (-4) and (-5): Change "$V_{1,2}$" to "V_1" and change "$V_{2,1}$" to "V_2".
- Page 220, line (-5): Change "γ" to "δ".
- Page 221, line (-1): Change "$\int \gamma$" to "$\int_\gamma$".
- Page 228, diagram (6.11): Change the last "X" to "Y".
- Page 233, line (-3): Change "characteristic" to "minimal".
- Page 235, paragraph starting "Again suppose": Also A and $\mathbf{k}$ are assumed to be structurally compatible as needed.
- Page 239, second display: Change "g" to "g_i" on the right side of the equality.

Chapter 7

- Page 283, lines 8–10: Replace the sentence beginning, "A complementary argument…" with, "For each $\overline{v} \in (\mathbf{Z}/N\mathbf{Z})^2$, the function $f_0^{\pm\overline{v}}$ determines two N-torsion points of E_j unless $2v = 0$, in which case it determines one (Exercise 7.5.5(a)), and so we have found all N^2 points of $E_j[N]$ (Exercise 7.5.5(b))."
- Page 286: Replace Exercise 7.5.5. The new exercise is, "(a) Show that for each $\overline{v} \in (\mathbf{Z}/N\mathbf{Z})^2$, the function $f_0^{\pm\overline{v}}$ determines two N-torsion points of E_j unless $2v = 0$, in which case it determines one. (b) Show that consequently, regardless of whether N is odd or even, we have found all N^2 points of $E_j[N]$."

- Page 290: In the second paragraph of Section 7.7, change "three" to "two" and remove the references to $\mathbf{K}_0'$, $\mathbf{C}(j, j_N)$, and K_0'. (It takes some work to show that $\mathbf{K}_0'$ is an intermediate field as claimed, and we do not need this result.)
- Page 291, line 14: Change "indeterminants" to "indeterminates".
- Page 294, line (−9): Change "K_0, K_0', and K_1" to "K_0 and K_1".

Chapter 8

- Page 316, line (−15): Change "lie in $\mathbf{k}$." to "lie in $\mathbf{k}$. For $\mathrm{char}(\mathbf{k}) = 2$, assume that every element of $\mathbf{k}$ is a square.".
- Page 326, line 1: Change the initial value "$a_1(E) = 1$" to "$a_1(E) = 2$". Furthermore, the normalized solution-counts that are denoted $a_{p^e}(E)$ on page 325 should be given a different name, as the true $a_{p^e}(E)$ are indeed defined as on page 361 by the same initial value and recurrence as the Fourier coefficients $a_{p^e}(f)$ of a newform. For now the normalized solution-counts are renamed $t_{p^e}(E)$. Note that $t_p(E) = a_p(E)$.
- Page 351, lines 4–6 (clarification, not correction): The argument given is necessary. The fact that the bottom arrow of the diagram is the zero map does not immediately show that $\ker(\tilde{\psi}) = \widetilde{E'}[p]$, because the domain of $\tilde{\psi}$ is all of $\widetilde{E'}$.
- Page 353, line (−8): Change "(7.18)" to "(7.18) (page 304)".
- Page 361: The right idea is to define for any prime p the local counting zeta-function of E, encoding the normalized solution-counts $t_{p^e}(E) = p^e + 1 - |\widetilde{E}(\mathbf{F}_{p^e})|$, as

$$Z_p(X, E) = \exp\left(\sum_{e=1}^{\infty} \frac{t_{p^e}(E)}{e} X^e\right).$$

Taking logarithmic derivatives shows that in fact for $X = p^{-s}$ the local zeta-function takes the form of an Euler factor,

$$Z_p(p^{-s}, E) = (1 - a_p(E)p^{-s} + \mathbf{1}_E(p)p^{1-2s})^{-1}.$$

The *Hasse-Weil L-function of E* is the product of these Euler factors,

$$L(s, E) = \prod_p (1 - a_p(E)p^{-s} + \mathbf{1}_E(p)p^{1-2s})^{-1}.$$

By the methods of the proof of Theorem 5.9.2, the Dirichlet series form of the L-function is

$$L(s, E) = \sum_{n=1}^{\infty} a_n(E)n^{-s}$$

where similarly to the Fourier coefficients of a newform, the $a_n(E)$ satisfy

$$\begin{aligned} a_1(E) &= 1, \\ a_p(E) &= p + 1 - |\widetilde{E}(\mathbf{F}_p)|, \\ a_{p^e}(E) &= a_p(E)a_{p^{e-1}}(E) - \mathbf{1}_E(p)pa_{p^{e-2}}(E), \quad e \geq 2, \\ a_{mn}(E) &= a_m(E)a_n(E), \quad (m,n) = 1, \end{aligned}$$

Chapter 9

- Page 368, line 16: Change "ramify in $\mathbf{Q}(\mu_N)$." to "ramify in $\mathbf{Q}(\mu_N)$ (except that 2 does not ramify if $N \equiv 2 \pmod 4$, but then $\mathbf{Q}(\mu_N) = \mathbf{Q}(\mu_{N/2})$ and $N/2$ is odd)."
- Page 385, exercise 9.4.2: Change the initial value from "$a_1(E) = 1$" to "$a_1(E) = 2$". Add the sentence, "Note that the equality holds for $e = 0$ as well." to the end of part (a). Change "Show that" to "Show that for $e \geq 2$" in part (b). Furthermore, this exercise applies to the $t_{p^e}(E)$ rather than to the $a_{p^e}(E)$, and so this change should be made throughout.
  Also, add to exercise 9.4.2: "(d) Again assume that $p \nmid N$. Show that

  $$(1 - a_p(E)x + px^2)^{-1} = (1 - \lambda_1 x)^{-1}(1 - \lambda_2 x)^{-1} = \sum_{e=0}^{\infty} \left(\sum_{c+d=e} \lambda_1^c \lambda_2^d \right) x^e.$$

  Explain why it follows that whereas the normalized prime-power solution-counts of the elliptic curve are $t_{p^e}(E) = \lambda_1^e + \lambda_2^e$, the corresponding prime-power Dirichlet coefficients of $L(s,E)$ are $a_{p^e}(E) = \sum_{c+d=e} \lambda_1^c \lambda_2^d$."

Hints and Answers to the Exercises

- Page 414, hint to Exercise 7.7.1: Remove "$j_N = j(E_j/\langle Q_\tau \rangle)$ and since", remove "$j_N^\sigma = j(E_j/\langle Q_\tau^\sigma \rangle)$ and", and change "In both cases the" to "The".