A First Course in Modular Forms: Corrections to the Fourth Printing

January 16, 2018

Chapter 1

- Page 7, line 8: Change "*the modular invariant*" to "the *modular invariant*".
- Page 10: Part (e) can be added to Exercise 1.1.7 as follows. "The next section will show that $(2\pi)^{-12}\varDelta$ has integral Fourier coefficients, so for the duration of this exercise, renormalize the discriminant function $\varDelta$ so that its first Fourier coefficient is 1. The integer-valued Fourier coefficient function of $\varDelta$ is called Ramanujan's τ-function. Because the variable of $\varDelta$ is also denoted τ and both usages of τ are time-honored, we accept the notation-collision

$$\varDelta(\tau) = \sum_{n=1}^{\infty} \tau(n) q^n.$$

Later in this book we will show that the cusp form space $\mathcal{S}_{12}(\mathrm{SL}_2(\mathbb{Z}))$ is one-dimensional. Thus $E_{12} - cE_6^2 = c\varDelta$ for some $c \in \mathbb{C}$. Show that $c = 24(1/B_6 - 1/B_{12})$. In light of the known value $B_{12} = -691/2730$ and the value $B_6 = 1/42$ from above, we thus have $-24 \cdot 691/B_{12} \in \mathbb{Z}$ and $691c \in -24 \cdot 691/B_{12} + 691\mathbb{Z}$. Explain why the Fourier coefficients of $691(E_{12} - 1)$ and $(-24 \cdot 691/B_{12})\varDelta$ are integers that are congruent modulo 691. Deduce Ramanujan's congruence,

$$\tau(n) \equiv \sigma_{11}(n) \pmod{691}, \quad n \geq 1."$$

Chapter 2

- Page 56, line 4: Change "are a subset" to "form a subset".

Chapter 3

- Page 80: In the "Conversely..." sentence immediately after the whitespace, change "$\omega(f)$" to "$\omega = \omega(f)$".
- Page 87: Change " for $k \geq 4$" to ", even $k \geq 4$" in (3.12).

- Page 88: Exercise 3.5.1 repeats Exercise 3.2.4.
- Page 91: Incidentally Theorem 3.6.1 shows that $\varepsilon_\infty^{\rm reg}$ is even.
- Page 92: Add a hint prompt to Exercise 3.6.4.

Chapter 4

- In section 4.8 and Exercise 4.8.6, the statement $G_1^{\overline{(-c_v,d_v)}}(\tau) = -G_1^{\overline{(c_v,d_v)}}(\tau)$ when $\bar{c}_v \neq 0$ is wrong; rather, $G_1^{\overline{(-c_v,-d_v)}}(\tau) = -G_1^{\overline{(c_v,d_v)}}(\tau)$ regardless of whether $\bar{c}_v = 0$; because the two vectors $\overline{\pm(c_v,d_v)}$ describe the same cusp of $\Gamma(N)$, these relations among the Eisenstein series do not support the claimed dimension $\varepsilon_\infty/2$ of the Eisenstein space $\mathcal{E}_1(\Gamma(N))$. See the paper *Moduli interpretation of Eisenstein series* by Kamal Khuri-Makdisi (International Journal of Number Theory, vol. 8, no. 3, 2012, pages 715–748), especially Proposition 4.3, for the subtle symmetry of weight 1 Eisenstein series.

Chapter 5

- Page 170, line 1: The first complete sentence should start, "Thus for any $\gamma \in \Gamma_0(N)\dots$".

Chapter 7

- Page 299, line (-2): Change $\tau_{P+P'}$ to $\tau^*_{P+P'}$.

Chapter 8

- Page 347: Change $h_{(0)}$ to h in the last display.

Chapter 9

- Page 371: Near the bottom of the page, change "an elliptic curve" to "an elliptic curve E", and change "a modular curve" to "a modular curve X".
- Page 379: The fifth line of section 9.2 should say "the points of order dividing ℓ^n".

Hints and Answers to the Exercises

- Page 417, line 2: Change E_4 to G_4.
- Page 417: Add a hint to Exercise 3.6.4, as follows. "For $k = 1$, the divisor $\mathrm{div}(\omega)$ is canonical and $\lfloor \mathrm{div}(f) \rfloor = (1/2)\mathrm{div}(\omega) + \sum_i (1/2)x_i$ and $\lfloor \mathrm{div}(f) - \sum_i x_i - \sum_i (1/2)x_i' \rfloor = (1/2)\mathrm{div}(\omega) - \sum_i (1/2)x_i$. Use the Riemann–Roch Theorem and its corollary."
- Page 428, line (-2): Change 0_E to 0.
- Page 431, solution to 9.3.2: Change "for some maximal ideal of $\mathcal{O}_\mathbb{F}$" to "for some maximal ideal $\mathfrak{p}_\mathbb{F}$ of $\mathcal{O}_\mathbb{F}$".

Index

- Page 444: Add "reduction of an elliptic curve over $\mathbb{Q}$, bad, semistable, 327" to the index.
- Page 445: Add "semistable reduction of an elliptic curve over $\mathbb{Q}$, 327" to the index.