DEFICIENCIES IN THE DIRICHLET ARITHMETIC PROGRESSION WRITEUP

• Insufficient analytic detail about Dirichlet series. The pending writeup on the class number formula for imaginary quadratic fields is more thorough in this context.

• Ad hoc continuation of \( \zeta(s) \) to \( 0 < \Re(s) < 1 \). The auxiliary writeup on continuations and functional equations for \( \zeta(s) \) and \( L(s, \chi) \) addresses this point, and the next one. The auxiliary writeup on local factors of zeta functions is also relevant.

• Conditional convergence of \( L(s, \chi) \) on \( 0 < \Re(s) < 1 \). See the comment in the previous bullet.

• Unmotivated, fragile argument that \( L(1, \chi) \neq 0 \). The auxiliary writeup on the cyclotomic zeta function partly addresses the motivation, but not the fragility. The writeup on a quadratic zeta function discusses some related ideas—both zeta functions are examples of Dedekind zeta functions, to play a key role in the class number formula for imaginary quadratic fields. The writeup on \( L(1, \chi) \) is less fragile but again unmotivated. A motivated, robust treatment of the issue is beyond our scope.

• What to do next? The writeup about Gaussian primes in sectors sketches the beginning of a vast generalization of Dirichlet’s Theorem on primes in arithmetic progressions.