

MATHEMATICS 311: COMPLEX ANALYSIS — ASSIGNMENT 7

**Reading:** Marsden, 4.1, 4.2, 4.3, 6.2, 6.3.

**Problems:**

1. Evaluate the following integrals by using residue techniques:

(a)  $\int_{-\infty}^{\infty} \frac{x^2 dx}{x^4 + 5x^2 + 6},$

(b)  $\int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)^3}$  where  $0 < a \in \mathbf{R},$

(c)  $\int_0^{2\pi} \frac{dx}{a + b \sin x}$  where  $a, b \in \mathbf{R}$  and  $a > |b| > 0,$

(d)  $\int_0^{\infty} \frac{\cos ax dx}{1 + x^2}$  where  $0 < a \in \mathbf{R},$

(e)  $\int_0^{\infty} \frac{\sqrt[3]{x} dx}{1 + x^2}.$

2. Let  $f(z)$  be analytic inside and on an ellipse  $\gamma$  with foci at  $\{-1, 1\}$ . Show that

$$\frac{z+1}{z-1} \in [-\infty, 0] \iff z \in [-1, 1].$$

Consequently, one can define  $\log\left(\frac{z+1}{z-1}\right)$  as a single-valued function on  $\gamma$ . Show that

$$\int_{\gamma} f(z) \log\left(\frac{z+1}{z-1}\right) dz = 2\pi i \int_{x=-1}^1 f(x) dx$$

where the right hand expression is a real integral, i.e.,  $x$  runs from  $-1$  to  $1$  in  $\mathbf{R}$ . (Begin by deforming the path of integration. The Residue Theorem plays no role in this problem.)

3. How many roots of the equation  $z^4 - 6z + 3 = 0$  have their modulus between 1 and 2?

4. If  $a \in \mathbf{R}$  and  $a > e$ , show that the equation  $e^z = az^n$  has  $n$  solutions in the disk  $\{z : |z| < 1\}$ .