MATHEMATICS 311: COMPLEX ANALYSIS — ASSIGNMENT 6

Reading: Marsden, section 3.3.

Problems: 1. Let $f(z)$ be analytic in the entire complex plane $\mathbb{C}$, and set $M(r) = \sup_{|z|=r} |f(z)|$. Show that $M(r_1) \le M(r_2)$ whenever $r_1 < r_2$, and determine when equality can occur.

2. Find all Laurent expansions in powers of z of the function

$$f(z) = \frac{1}{z^2(z^2+1)(z^2+9)}.$$

(Note that z^2 is already a power of z. Use partial fractions and then the handy formulas from class.)

3. Find all Laurent expansions in powers of $z-1$ and in powers of $z+1/2$ of the function

$$f(z) = \frac{1}{(2z+1)(z-1)}.$$

(There is no need to use partial fractions on this problem.)

4. Suppose $f(z)$ is analytic in the strip $a < y < b$ (where $z = x+iy$) and satisfies $f(z+1) = f(z)$. Show that $f(z)$ has a complex Fourier expansion

$$f(z) = \sum_{n=-\infty}^{\infty} a_n e^{2\pi i n z}$$

converging at all points of this strip, where

$$a_n = e^{\pi n(a+b)} \int_{x=0}^{1} e^{-2\pi i n x} f(x + \frac{a+b}{2} i)\,\mathrm{d}x.$$

(Suggestion: Consider the mapping $w = e^{2\pi i z}$. Show that $f(z)$ determines an analytic function of w in the appropriate region, and use the Laurent expansion. Indicate where the hypothesis $f(z+1) = f(z)$ is needed.)

5. Find all singular points (either in the finite complex plane $\mathbb{C}$ or at ∞) of the following functions, and for each one indicate whether it is nonisolated, essential, a pole, or removable.

(a) $f(z) = 1/[(2z+1)(z-1)]$;

(b) $f(z) = 1/(e^z - 1)$;

(c) $f(z) = \pi/\sin(\pi/z)$.