Reading: Marsden, section 3.3.

Problems: 1. Let \( f(z) \) be analytic in the entire complex plane \( \mathbb{C} \), and set \( M(r) = \sup_{|z|=r} |f(z)| \). Show that \( M(r_1) \leq M(r_2) \) whenever \( r_1 < r_2 \), and determine when equality can occur.

2. Find all expansions in powers of \( z \) of the function
\[
f(z) = \frac{1}{z^2(z^2 + 1)(z^2 + 9)}.
\]
(Note that \( z^2 \) is already a power of \( z \). Use partial fractions and then the handy formulas from class.)

3. Find all expansions in powers of \( z - 1 \) and in powers of \( z + 1/2 \) of the function
\[
f(z) = \frac{1}{(2z + 1)(z - 1)}.
\]
(There is no need to use partial fractions on this problem.)

4. Suppose \( f(z) \) is analytic in the strip \( a < y < b \) (where \( z = x + iy \)) and satisfies \( f(z + 1) = f(z) \). Show that \( f(z) \) has a complex Fourier expansion
\[
f(z) = \sum_{n=-\infty}^{\infty} a_n e^{2\pi inz}
\]
converging at all points of this strip, where
\[
a_n = e^{\pi n(a+b)} \int_{x=0}^{1} e^{-2\pi inx} f(x + \frac{a + b}{2} - it) \, dx.
\]
(Suggestion: Consider the mapping \( w = e^{2\pi iz} \). Show that \( f(z) \) determines an analytic function of \( w \) in the appropriate region, and use the two-sided expansion. Indicate where the hypothesis \( f(z + 1) = f(z) \) is needed.)

5. Find all singular points (either in the finite complex plane \( \mathbb{C} \) or at \( \infty \)) of the following functions, and for each one indicate whether it is nonisolated, essential, a pole, or removable.
   (a) \( f(z) = 1/[(2z + 1)(z - 1)] \);
   (b) \( f(z) = 1/(e^z - 1) \);
   (c) \( f(z) = \pi/\sin(\pi/z) \).