MATHEMATICS 311: COMPLEX ANALYSIS — ASSIGNMENT 4

Reading: Marsden, sections 2.4, 2.5.

Problems:

1. Evaluate $\int_\gamma \frac{ze^z}{z+2i}\,\mathrm{d}z$ in the following two cases: (a) $\gamma = \{z \in \mathbb{C} : |z| = 1\}$, (b) $\gamma = \{z \in \mathbb{C} : |z| = 3\}$.

2. Evaluate $\int_{|z|=1} e^z z^{-4}\,\mathrm{d}z$.

3. Show that for any complex number t,

$$\frac{1}{2\pi i}\int_{|z|=3} \frac{e^{zt}}{z^2+1}\,\mathrm{d}z = \sin t.$$

4. Prove Cauchy's inequality: If f is analytic in an open neighborhood of the closed disk $\{\zeta \in \mathbb{C} : |\zeta - z| \leq r\}$ and if f satisfies $|f(\zeta)| \leq M$ whenever $|\zeta - z| = r$ then $|f^{(n)}(z)|/n! \leq M/r^n$.

5. Show that if f is analytic in the entire plane $\mathbb{C}$, and for some positive real number c and some nonnegative integer n and some positive real number r_o we have $|f(z)| \leq c|z|^n$ for all z such that $|z| \geq r_o$, then f must be a polynomial of degree at most n. (Hint: Since f is represented everywhere by its power series about 0, it suffices to show that $f^{(n+m)}(0) = 0$, for all positive integers m, i.e., that $|f^{(n+m)}(0)|$ is arbitrarily small for any such m.)

6. Show that there can not exist any function f that is analytic in an open neighborhood of a point z and satisfies $|f^{(n)}(z)|/n! > n^n$ for all positive integers n.