**MATHEMATICS 311: COMPLEX ANALYSIS — ASSIGNMENT 4**

**Reading:** Marsden, sections 2.4, 2.5.

**Problems:**

1. Evaluate \( \int_{\gamma} \frac{ze^z}{z + 2i} \, dz \) in the following two cases: (a) \( \gamma = \{ z \in \mathbb{C} : |z| = 1 \} \), (b) \( \gamma = \{ z \in \mathbb{C} : |z| = 3 \} \).

2. Evaluate \( \int_{|z|=1} e^z z^{-4} \, dz \).

3. Show that for any complex number \( t \),
   \[
   \frac{1}{2\pi i} \int_{|z|=3} \frac{e^{zt}}{z^2 + 1} \, dz = \sin t.
   \]

4. Prove Cauchy’s inequality: If \( f \) is analytic in an open neighborhood of the closed disk \( \{ \zeta \in \mathbb{C} : |\zeta - z| \leq r \} \) and if \( f \) satisfies \(|f(\zeta)| \leq M \) whenever \(|\zeta - z| = r \) then \(|f^{(n)}(z)|/n! \leq M/r^n \).

5. Show that if \( f \) is analytic in the entire plane \( \mathbb{C} \), and for some positive real number \( c \) and some nonnegative integer \( n \) and some positive real number \( r_o \) we have \(|f(z)| \leq c|z|^n \) for all \( z \) such that \(|z| \geq r_o \), then \( f \) must be a polynomial of degree at most \( n \). (Hint: Since \( f \) is represented everywhere by its power series about 0, it suffices to show that \( f^{(n+m)}(0) = 0 \), for all positive integers \( m \), i.e., that \(|f^{(n+m)}(0)| \) is arbitrarily small for any such \( m \).)

6. Show that there cannot exist any function \( f \) that is analytic in an open neighborhood of a point \( z \) and satisfies \(|f^{(n)}(z)|/n! > n^n \) for all positive integers \( n \).