CALCULUS AND ANALYSIS IN EUCLIDEAN SPACE: ADDITIONS AND CORRECTIONS SECOND PRINTING

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As of autumn 2019, the second printing is available. To check whether a copy of the book is the second printing, see if it has the first few additions and corrections to the first printing.

Chapter 2

- Page 53: A variant proof of Proposition 2.4.7 is as follows: "Let $\{x_\nu\}$ converge to p. Note that $\{x_\nu\}$ is the translate of the null sequence $\{y_\nu\} = \{x_\nu - p\}$ by p. The triangle inequality shows that every translate of a bounded sequence is again bounded, and so it suffices to show that every null sequence $\{y_\nu\}$ is bounded. Given such a sequence, there exists a starting index ν_0 such that $|y_\nu| < 1$ for all $\nu > \nu_0$. For any real number $R > \max\{|y_1|, \dots, |y_{\nu_0}|, 1\}$, we have $\{y_\nu\} \subset B(\mathbf{0}, R)$ as a set. Thus $\{y_\nu\}$ is bounded, as desired."

Chapter 4

- Page 149: The proof of Lemma 4.4.4(1) is more tidily concluded by nothing that hk is a twofold product of linear functions and therefore is $o(h,k)$ by Proposition 4.2.6.
- Page 155, line 9: Change "$f_i a$)" to "$f_i(a)$".
- Page 165, beginning of five-line display: Change "$\dfrac{\mathrm{d}F(x,y)}{\mathrm{d}x}$" to "$\dfrac{\mathrm{d}F(x)}{\mathrm{d}x}$".

Chapter 5

- Page 212: In exercise 5.2.8(a), change the prompt to, "(Because this is a one-dimensional problem, you may verify the old definition of derivative rather than the new one; alternatively, note that $f = T + g$ where T is linear and $g(h)$ is $o(h)$.)"

Chapter 6

- Page 280: A tider proof of Theorem 6.4.3 is, "Define $F_2 : [a,b] \longrightarrow \mathbb{R}$ by $F_2(x) = \int_a^x F'$. Then $F_2' = F'$ by the preceding theorem, so (Exercise 6.4.3) there exists a constant c such that $F_2(x) = F(x) + c$ for all $x \in [a,b]$. Thus, because $F_2(a) = 0$,

$$\int_a^b F' = F_2(b) - F_2(a) = F(b) + c - F(a) - c = F(b) - F(a)."$$

- Page 282: Replace the paragraph after Corollary 6.4.4 by, "The formula in the corollary is the formula for **integration by inverse substitution**. To obtain it from (6.4), substitute $f \cdot (\phi^{-1})'$ for f in (6.4) to get

$$\int_a^b (f \circ \phi) \cdot ((\phi^{-1})' \circ \phi) \cdot \phi' = \int_{\phi(a)}^{\phi(b)} f \cdot (\phi^{-1})'.$$

By the chain rule, $((\phi^{-1})' \circ \phi) \cdot \phi' = (\phi^{-1} \circ \phi)' = 1$, and so we have the result."

- Page 313, fourth display: "$R_{\geq 0}$" should be "$\mathbb{R}_{\geq 0}$".
- Exercise 6.7.15: Add part (d) as follows. "(d) Use various exercises from this section to show that the centroid of the upper half n-ball has last coordinate
$$\overline{x}_n = \frac{2^{n+1}}{\pi(n+1)} \cdot \frac{(n/2)!^2}{n!},$$
and then use Stirling's formula to show that asymptotically, as n grows,
$$\overline{x}_n \sim \sqrt{\frac{2}{\pi n}}.$$
Thus the n-ball is concentrated ever more toward its center."
- Section 6.8: This section can be omitted if we strengthen the hypotheses of the change of variable theorem to assume that $\Phi(K)$ has boundary of volume zero. For all 2-dimensional and 3-dimensional examples in a calculus class, one can see this to be the case.
- Page 328, line (−3): Change "that that" to "that".

Chapter 9

- Page 443, line (−4): Change "consisted only in" to "consisted only of".
- Page 459: A better end to the proof of Theorem 9.10.1 is

$$\begin{aligned}
\int_{\Phi} f\,\mathrm{d}x_I &= \int_D (f \circ \Phi) \det \Phi'_I && \text{by definition, as in (9.14)} \\
&= \int_{\Delta^D} (f \circ \Phi) \det \Phi'_I \,\mathrm{d}u_1 \wedge \cdots \wedge \mathrm{d}u_k && \text{by Exercise 9.5.4} \\
&= \int_{\Delta^D} \Phi^* f \cdot \Phi^* \mathrm{d}x_I && \text{by Theorem 9.9.3} \\
&= \int_{\Delta^D} \Phi^*(f\,\mathrm{d}x_I) && \text{by Theorem 9.9.4(2).}
\end{aligned}$$