CALCULUS AND ANALYSIS IN EUCLIDEAN SPACE: CORRECTIONS

December 14, 2018

Chapter 1

- Page 16: On line 7, "polynomial growth" should be "exponential growth".

Chapter 3

- Page 91: If *some* determinant can be constructed without reference to the fact that every permutation has a well defined sign (i.e., every sequence of transpositions that unscrambles it has the same parity), then the argument on this page shows that indeed the sign of every permutation is well defined in consequence of some determinant existing. Further, as shown later in the chapter, any determinant formula must take the form

$$\det(A) = \sum_{\pi \in S_n} (-1)^{\pi} a_{1\pi(1)} a_{2\pi(2)} \cdots a_{n\pi(n)}$$

for *some* sign function on permutations (i.e., assigning to each permutation one of ± 1 according to the parity of the length of a particular chosen sequence of transpositions that unscrambles it); that is, there are as many candidate determinant functions as there are sign functions. Now, constructing one determinant, either with a sign function that may not be unique, or without a sign function at all, shows that the sign function is unique, and therefore the determinant is unique in consequence.

The text uses a particular sign function to construct a determinant, and then the discussion here shows that the sign function used in the text is the only possible sign function and the determinant is unique. Alternatively, the standard inductive determinant construction with no reference to a sign function at all is $\det_1([a]) = a$ and then, letting A^{1j} denote the $(n-1) \times (n-1)$ matrix obtained by removing the first row and jth column of A,

$$\det_n(A) = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det_{n-1}(A^{1j}), \quad n \geq 2.$$

One can start instead with $\det_0([\,]) = 1$ and then the displayed formula for $n \geq 1$.

- Page 114: The discussion can be improved, e.g., "As for boxes, scaling any spanning vector of a parallelepiped by a real number a magnifies the volume by $|a|$, and so we have

$$\text{vol } T\mathcal{B}' = \text{vol } T\mathcal{B} \cdot a_1 \cdots a_n.$$

But also,

$$a_1 \cdots a_n = \text{vol } \mathcal{B}'.$$

That is, the volume of the T-image of any box is a constant multiple of the volume of the box, regardless of the box's location or side lengths, the constant being the volume of $T\mathcal{B}$, the T-image of the unit box $\mathcal{B}$. Call this constant magnification factor t. Thus,

$$\text{vol}\, T\mathcal{B}' = t \cdot \text{vol}\, \mathcal{B}' \quad \text{for all boxes } \mathcal{B}'\text{."}$$

Chapter 4

- Page 135: The second display should say "$\psi(h) = h_i$", not "$\varphi(h) = h_i$".
- Page 145: Proposition 4.3.4 should be titled "Differentiability implies continuity".
- The insistence on the formula $f'' = (f'^{\mathsf{T}})'$ in section 4.7 is silly. Simply view f' as a mapping to $\mathbb{R}^n$, each of whose outputs is an ordered list rather than specifically a row vector.
- Page 152: A better version of exercise 4.4.8(a) is: "Show that if f is multilinear and $a_1, \dots, a_k, h_1, \dots, h_k \in \mathbb{R}^n$ then for any $j \in \{2, \dots, k\}$, $f(h_1, \dots, h_j, a_{j+1} \dots, a_k)$ is $o(h_1, \dots, h_k)$. The same result holds if any j inputs to f are h's, rather than the first j inputs, because permuting the inputs of a multilinear function creates another multilinear function. Flesh this argument out as much as feels necessary for your understanding."
- Page 182: Change "G" to "T" in the first line after the second display.
- Page 188: The left side of Figure 4.12 has different horizontal and vertical scales; scaled correctly, it would show the two vectors at right angles.

Chapter 5

- Page 206: The sentence after the third display could be expanded to, "This formula combines with Proposition 4.3.4 (differentiability implies continuity) and Corollary 3.7.3 (the entries of the inverse matrix are continuous functions of the entries of the matrix) to show that since the mapping is continuously differentiable and the local inverse is differentiable, the local inverse is continuously differentiable: If y varies slightly, then so does x because f^{-1} is continuous, hence so does $f'(x)$ because f' is continuous, hence so does $f'(x)^{-1}$, which is $(f^{-1})'(y)$."

Chapter 6

- Page 268: Exercise 6.2.7 should say that upper sums for f can be bigger than *lower* sums for g.
- Page 295: In the statement of Fubini's theorem, expand the "Suppose..." sentence to, "Suppose that for each $x \in [a, b]$, the cross-sectional integral $\int_{y=c}^{d} f(x, y)$ exists; this happens if the cross-sectional function $\varphi_x : [c, d] \longrightarrow \mathbb{R}$ given by $\varphi_x(y) = f(x, y)$ is continuous as a function of y except on a subset of length zero, and in particular this happens if S contains only finitely many points (possibly none) having first coordinate x." This expanded statement covers cases such as $f : [0, 2] \times [0, 1] \longrightarrow \mathbb{R}$ where $f(x, y) = 0$ if $x \leq 1$ and $f(x, y) = 1$ if $x > 1$, with the constant function $\varphi_1 = 0$ integrable along the cross-section at $x = 1$ even though this cross-section is the discontinuity set S of f.
- Page 305: Quoting Theorem 6.4.1 requires showing that $\int_{y=c}^{d} D_1 f(t, y)$ is a continuous function of t. Fix t, and let $\varepsilon > 0$ be given. The continuity of $D_1 f$ on $[a, b] \times [c, d]$ is uniform, so for some $\delta > 0$, for all $\tilde{t}$ such that $|\tilde{t} - t| < \delta$, we have $|D_1 f(\tilde{t}, y) - D_1 f(t, y)| < \varepsilon/(d - c)$ for all $y \in [c, d]$.

Thus for all such $\tilde{t}$,

$$\left|\int_{y=c}^{d} D_1 f(\tilde{t}, y) - \int_{y=c}^{d} D_1 f(t, y)\right| \leq \int_{y=c}^{d} |D_1 f(\tilde{t}, y) - D_1 f(t, y)| < \varepsilon.$$

Chapter 9

- Page 448: In the "...(now dropping the wedges from the notation)..." sentence, drop the wedge from the notation $\mathrm{d}x \wedge \mathrm{d}y$.
- Page 474: $\Delta^3_{1,b}$ should be $\Delta^3_{3,b}$ in two places toward the bottom of the page.
- Page 476: As the text explains, there is no reason to compute that the double boundary operator is zero, but nonetheless an exercise to do so as as follows.

  "This exercise gives a self-contained proof that the double boundary operator is identically zero. It suffices to show this for the double boundary on the standard k-cube, where $k \geq 2$.

  (a) Explain why the double boundary is

$$\partial^2 \Delta^k = \sum_{i=1}^{k} \sum_{\alpha=0}^{1} \sum_{j=1}^{k-1} \sum_{\beta=0}^{1} (-1)^{i+j+\alpha+\beta} \Delta^k_{i,\alpha} \circ \Delta^{k-1}_{j,\beta}.$$

  (b) Show that if $i \leq j$ then we have

$$\Delta^k_{i,\alpha} \circ \Delta^{k-1}_{j,\beta}(u_1, \ldots, u_{k-2}) = (u_1, \ldots, u_{i-1}, \alpha, u_i, \ldots, u_{j-1}, \beta, u_j, \ldots, u_{k-2}),$$

  with α in the ith slot and β in the $(j+1)$st slot, whereas if $i > j$ then we have

$$\Delta^k_{i,\alpha} \circ \Delta^{k-1}_{j,\beta}(u_1, \ldots, u_{k-2})(u_1, \ldots, u_{j-1}, \beta, u_j, \ldots, u_{i-2}, \alpha, u_{i-1}, \ldots, u_{k-2}),$$

  with β in the jth slot and α in the ith slot. Thus the double boundary of the standard k-cube consists of two sums, written as formal sums of functions of the variables $u_1, \ldots, u_{k-2}$,

$$\begin{aligned}\partial^2 \Delta = &\sum_{i=1}^{k-1} \sum_{j=i}^{k-1} (-1)^{i+j+\alpha+\beta} (u_1, \ldots, u_{i-1}, \alpha, u_i, \ldots, u_{j-1}, \beta, u_j, \ldots, u_{k-2}) \\ &+ \sum_{i=1}^{k} \sum_{j=1}^{i-1} (-1)^{i+j+\alpha+\beta} (u_1, \ldots, u_{j-1}, \beta, u_j, \ldots, u_{i-2}, \alpha, u_{i-1}, \ldots, u_{k-2}).\end{aligned}$$

  (c) Explain why the second double sum can instead be written as

$$\sum_{j=1}^{k-1} \sum_{i=j+1}^{k} (-1)^{i+j+\alpha+\beta} (u_1, \ldots, u_{j-1}, \beta, u_j, \ldots, u_{i-2}, \alpha, u_{i-1}, \ldots, u_{k-2}).$$

  (d) Convince yourself that it is valid to replace i by $i+1$ in this new second sum, and that doing so gives

$$-\sum_{j=1}^{k-1} \sum_{i=j}^{k-1} (-1)^{i+j+\alpha+\beta} (u_1, \ldots, u_{j-1}, \beta, u_j, \ldots, u_{i-1}, \alpha, u_i, \ldots, u_{k-2}),$$

  now with α in the $(i+1)$st slot.

(e) Convince yourself that it is valid to exchange the roles of i and j, and to exchange the roles of α and β, and that doing so gives

$$-\sum_{i=1}^{k-1}\sum_{j=i}^{k-1}(-1)^{i+j+\alpha+\beta}(u_1,\dots,u_{i-1},\alpha,u_i,\dots,u_{j-1},\beta,u_j,\dots,u_{k-2}).$$

This cancels the first sum, so we are done."

- Page 480: In the first paragraph after the proof ends, change "of k-surfaces" to "of k-surfaces and their boundaries".