CALCULUS AND ANALYSIS IN EUCLIDEAN SPACE:  
CORRECTIONS

October 27, 2018

Chapter 1

• Page 16: On line 7, “polynomial growth” should be “exponential growth”.

Chapter 3

• Page 114: The discussion can be improved, e.g., “As for boxes, scaling any spanning vector of a parallelepiped by a real number \(a\) magnifies the volume by \(|a|\), and so we have

\[
\text{vol } TB' = \text{vol } TB \cdot a_1 \cdots a_n.
\]

But also,

\[
a_1 \cdots a_n = \text{vol } B'.
\]

That is, the volume of the \(T\)-image of any box is a constant multiple of the volume of the box, regardless of the box’s location or side lengths, the constant being the volume of \(TB\), the \(T\)-image of the unit box \(B\). Call this constant magnification factor \(t\). Thus,

\[
\text{vol } TB' = t \cdot \text{vol } B' \text{ for all boxes } B'.
\]

Chapter 4

• Page 135: The second display should say “\(\psi(h) = h_i\)”, not “\(\varphi(h) = h_i\)”.

• Page 145: Proposition 4.3.4 should be titled “Differentiability implies continuity”.

• The insistence on the formula \(f'' = (f'^T)'\) in section 4.7 is silly. Simply view \(f'\) as a mapping to \(\mathbb{R}^n\), each of whose outputs is an ordered list rather than specifically a row vector.

• Page 152: A better version of exercise 4.4.8(a) is: “Show that if \(f\) is multilinear and \(a_1, \ldots, a_k, h_1, \ldots, h_k \in \mathbb{R}^n\) then for any \(j \in \{2, \ldots, k\}\), \(f(h_1, \ldots, h_j, a_{j+1}, \ldots, a_k) = o(h_1, \ldots, h_k)\). The same result holds if any \(j\) inputs to \(f\) are \(h\)’s, rather than the first \(j\) inputs, because permuting the inputs of a multilinear function creates another multilinear function. Flesh this argument out as much as feels necessary for your understanding.”

• Page 182: Change “\(G\)” to “\(T\)” in the first line after the second display.

• Page 188: The left side of Figure 4.12 has different horizontal and vertical scales; scaled correctly, it would show the two vectors at right angles.

Chapter 5

• Page 206: The sentence after the third display could be expanded to, “This formula combines with Proposition 4.3.4 (differentiability implies continuity) and Corollary 3.7.3 (the entries of the inverse matrix are continuous functions of the entries of the matrix) to show that since the mapping is continuously differentiable and the local inverse is differentiable, the local inverse is continuously differentiable: If \(y\) varies slightly, then so does \(x\).
because \(f^{-1}\) is continuous, hence so does \(f'(x)\) because \(f'\) is continuous, hence so does \(f'(x)^{-1}\), which is \((f^{-1})'(y)\).

**Chapter 6**

- Page 268: Exercise 6.2.7 should say that upper sums for \(f\) can be bigger than lower sums for \(g\).
- Page 295: In the statement of Fubini’s theorem, expand the “Suppose…” sentence to, “Suppose that for each \(x \in [a, b]\), the cross-sectional integral \(\int_{y=c}^{d} f(x, y)\) exists; this happens if the cross-sectional function \(\varphi_x : [c, d] \to \mathbb{R}\) given by \(\varphi_x(y) = f(x, y)\) is continuous as a function of \(y\) except on a subset of length zero, and in particular this happens if \(S\) contains only finitely many points (possibly none) having first coordinate \(x\).” This expanded statement covers cases such as \(f : [0, 2] \times [0, 1] \to \mathbb{R}\) where \(f(x, y) = 0\) if \(x \leq 1\) and \(f(x, y) = 1\) if \(x > 1\), with the constant function \(\varphi_1 = 0\) integrable along the cross-section at \(x = 1\) even though this cross-section is the discontinuity set \(S\) of \(f\).
- Page 305: Quoting Theorem 6.4.1 requires showing that \(\int_{y=c}^{d} D_1 f(t, y)\) is a continuous function of \(t\). Fix \(t\), and let \(\varepsilon > 0\) be given. The continuity of \(D_1 f\) on \([a, b] \times [c, d]\) is uniform, so for some \(\delta > 0\), for all \(\tilde{t}\) such that \(|\tilde{t} - t| < \delta\), we have \(|D_1 f(\tilde{t}, y) - D_1 f(t, y)| < \varepsilon/(d - c)\) for all \(y \in [c, d]\). Thus for all such \(\tilde{t}\),

\[
\left| \int_{y=c}^{d} D_1 f(\tilde{t}, y) - \int_{y=c}^{d} D_1 f(t, y) \right| \leq \int_{y=c}^{d} |D_1 f(\tilde{t}, y) - D_1 f(t, y)| < \varepsilon.
\]

**Chapter 9**

- Page 448: In the “…(now dropping the wedges from the notation)…” sentence, drop the wedge from the notation \(dx \wedge dy\).
- Page 454: In the six-line display, change “recognizing the definition of \(d\)” to “by the product rule for \(d\), nilpotence” at the fifth line.
- Page 474: \(\Delta_{3, b}^3\) should be \(\Delta_{3, b}^3\) in two places toward the bottom of the page.