How to be an Atomist

Paul Hovda

November 18, 2005

Contents

1 Extreme Atomism

1.1 What is Atomism?

Say that an "atom" is a thing that has no parts (except itself). Atomism is the doctrine that:

> Everything is composed of some atoms.

There is a stronger doctrine, which I will call *Strict Atomism*. It adds something like the following to Atomism:

> All facts can be adequately characterized as facts involving nothing but atoms.

A still stronger doctrine is *Extreme Atomism*, an idea roughly stated as

> Everything (besides *abstracta* like sets) is an atom.

Among the ancients, Democritus may have been the first reflective Atomist, and was probably an Extreme Atomist; Epicurus was an Atomist, but certainly not an Extreme Atomist, and perhaps not even a Strict Atomist.[1] Among contemporaries, many philosophers seem attracted to Atomism, and many to Strict Atomism (depending, of course, on just what is meant by "adequately characterized" and "fact about"). (Many, for example, would at least take seriously the claim that all facts about non-atoms *supervene on* facts about atoms.)

Few, however, express a clear commitment to Extreme Atomism. Peter van Inwagen entertains the position extensively, under the name 'Nihilism', but does not advocate it, in *Material Beings*.

---

[1]Both believed in void as well as body, and Democritus seemed to allow that atoms have different shapes, which suggests that atoms have extent and hence occupy a nonatomic place. Epicurus held that void (or place) itself is composed of indivisible parts, (*minima*), which are "atoms" in the sense of having no proper parts. Atomic bodies actually occupy some finite number of minima, typically, or maybe always, more than one. The minima of place are like "points of space" in that they are not composed of smaller places, but they are unlike "points of space" since there are adjacent minima (pairs of minima with no minimum between them). That Epicurus was not an extreme Atomist is evident from his recognition of a division among bodies between composite and atomic; he also insisted that perceptible qualities somehow exist. (See the *Letter to Herodotus*.)

I will argue that, given certain natural assumptions, the Extreme Atomist position can be thoroughly defended against one line of attack. The line of attack is, roughly, this:

> People use ordinary and scientific language to say all kinds of true things, and they appear to be (both singularly and plurally) quantifying over, referring to, and predicating properties and relations of, non-atomic concrete objects. There is no good way to explain this appearance except as a reality: we successfully refer to, etc. concrete objects, hence they, as well as atoms, exist.

One of the main reasons an Atomist might stop short of Extreme Atomism is that it would appear to make our ordinary and scientific talk and thought false and illusory.

I believe that a defense of Extreme Atomism can be given on which our talk and thought is only false and illusory along one dimension, and not through-and-through. The dimension is number. When we assert, for example,

> John is walking.

we take ourselves to refer to one thing, and to predicate a certain property of that thing. Our assertion is true or false, it might seem, according to whether or not that thing has the property in question. The Extreme Atomist can hold that the only illusion here lies in our idea that we have referred to some one thing. In fact, we referred to many things—the atoms that "make up John" so to speak. (But of course, the whole point is that there is not really any further thing, John, made up of the atoms.) Other than that, our ordinary picture is alright: our assertion is true or false according to whether or not *those atoms* collectively have the property expressed by the predicate '__ is walking'.

Similarly, when we assert

> Something is walking.

what we assert is true, but not because there is some genuinely *single* thing that has the property of walking; rather, it is because there are some atoms that have that property.

1.2   Logical difficulties with plurals

These are the beginnings of a line of thought on which ordinary (and scientific) sentences are supposed to be true and false as we would otherwise expect, but on which the only *objects* we ever refer to or quantify over are atoms. If it can be followed through, the Extreme Atomist can escape the charge of falsifying most things we take to be true. But it runs into certain logical and technical difficulties. For example, if 'John' refers to some atoms, and 'the atoms that make up John' also refers to those atoms, then these two expressions are co-referential, and hence ought to be inter-substitutable *salva veritate,* at least in non-epistemic (or otherwise "opaque") contexts. But they are not:

> Every one of the atoms that make up John is an atom.

is true, while

> Every one of John is an atom.

is false.[2] One can see here the pattern for pervasive difficulties for the Extreme Atomist's interpretive program, difficulties that arise in connection with plural constructions in natural language.

One of the central tasks of this paper is to show how logical difficulties like these, concerning the logical properties of identity and 'is one of', can be overcome without abandoning the Extreme Atomist's metaphysics.

Overall, the defense of Extreme Atomism will consist in displaying a thoroughly systematic way to interpret sentences of (a significant fragment of) ordinary and scientific language, so as to: (1) get the truth-conditions

---

[2]It's falseness does not depend on its containing a phrase of the form $\ulcorner$Every one of $x\urcorner$ with a grammatically singular term $\ulcorner x\urcorner$. (In fact, we should take it, as a limit case, that 'John is one of John' is a logical truth.) A similar issue arises if we consider the plural term 'John and Paul' and the term 'the atoms that make up John and Paul'. It appears that the atomist must say that these expressions are co-referential, while surely

> Every one of the atoms that make up John and Paul is an atom.

is true, while

> Every one of John and Paul is an atom.

is false.

right, (2) solving the kind of logical difficulties suggested above, (3) without appealing to anything other than atoms and properties of (and relations among) atoms. In particular, the entities appealed to (as subjects of predication in the account) will only be atoms, never composite objects or sets (or even properties). The defense will use plural quantification over atoms, the postulation of a host of properties of atoms, and quantification (plural quantification, even) over these properties and relations.

Roughly, the Atomist identifies a composite object with its atoms. We will try to answer the objection that this cannot be correct because of the apparent "numerical difference" between the one and the many. There are other apparent differences that we will not address. One might plausibly argue that we can see such differences when we look at *time*, and *modality*. For example, a thing might be composed of different atoms at different times; but some atoms cannot *be* different atoms at different times. And a composite object *might have been* composed of different atoms, but some atoms could not have been different atoms. The Atomist can answer these objections; answers to similar objections to the identification of a thing with the mereological sum of its parts can be used also by the Atomist. Very briefly, the first can be answered by incorporating a four-dimensionalist view of time and thinking of the atoms as instantaneous things and the composite as thus spread out in time; the second can be answered by incorporating a counterpart-theoretic treatment of modality, and invoking different counterpart relations for the apparent modal differences.

We will ignore these issues throughout the rest of the paper; the fragment of natural language that we interpret will not include temporal and modal language. To simplify further, it will include no epistemic operators, self-reference or indexicals, either. It will include (singular and plural) names, (singular and plural) basic predicates, and the apparatus of singular and plural quantification.

2 Preliminaries

Before giving the Atomist's interpretive scheme, we must consider some preliminary matters involving plural predication. In this section of the paper, we entirely set aside the issue of Atomism, and proceed as if 'John' refers to one thing, a man, and so forth.

First, we will argue for the claim that there is a systematic ambiguity attending plural predications in English: (unless explicitly disambiguated in context) all of them are ambiguous between a collective and a distributive reading. (Sometimes there are other readings, as well.)

Second, we will suggest that, given a metaphysics of properties and relations, and propositions formed by "predicating" properties and relations of individuals, it is natural to hold that any property can be predicated of one individual or many individuals (collectively) to form a proposition. In the latter case, such a proposition would naturally correspond to an English plural predication, read collectively (assuming that the English predicate expresses a property). Further, each "place" of an n-place relation can be "filled with" one or many things collectively, to form a proposition. Again, such propositions will naturally correspond with English sentences. Thus there is no fundamental distinction between "plural properties" and "singular properties".

2.1 Collective vs. distributive in English

Consider the sentence

> John, Paul, and George lifted the piano.

The sentence is ambiguous between two readings, commonly called the "distributive" and the "collective" readings. The distributive reading is brought out explicitly in the sentence

> John, Paul, and George each lifted the piano.

It can be seen as a disguised quantification with an embedded singular predication:

> Every thing that is one of John, Paul, and George is such that it lifted the piano.

This, in turn, appears to be logically equivalent with

> John lifted the piano and Paul lifted the piano and George lifted the piano.

The collective reading, on the other hand, is brought out explicitly by the sentence

> John, Paul, and George lifted the piano together.

Note that the distributive does not logically imply the collective reading; if each of them lifted the piano, they may or may not have lifted the piano together.

Does the collective imply the distributive? In this case, it is a bit hard to say. If it is only through their joint agency that they were able to lift the piano, then perhaps it is strictly false to say of any one that he lifted it. However it goes in this case, in general, the collective version of a plural predication *does not* imply the singular version.

Here is an example that shows this quite clearly.

> P and P → Q entail Q.

(I am using the symbols P and P → Q and Q as names of symbolic sentences of a formal propositional language, and I mean to be speaking of tautological entailment.) Here the collective reading of the sentence is true, but the distributive reading is false. It is not the case that *each of* P and P → Q entails Q. But the two together do.

One lesson to take away is that the two readings are generally, logically independent; neither implies the other. There may be logical connections that flow from the predicates: with the intransitive verb 'sang', the collective predication implies the distributive, and not vice-versa. With the relational predicate 'entails Q', the distributive implies the collective, but not vice-versa.

With relational expressions, each of the two *relata* spots is subject to the distributive/collective ambiguity. Hence

> The boys lifted the pianos.

could mean any of (at least) four things. Explicitly disambiguating leads to unusual and awkward-seeming English (unusual, since context usually allows one to "resolve the ambiguity"—to know what was intended). For example:

> The boys, collectively, lifted, collectively, the pianos.

Relative clauses make for deliciously ambiguous sentences like

> The pianos the boys lifted weigh 2000 pounds.

Explicitly disambiguating is difficult.

> The pianos the boys collectively lifted weigh, collectively, 2000 pounds.

is itself ambiguous—is this about the pianos such that each of them was lifted by the boys collectively, or is it about some pianos such that each boy lifted them collectively? (Below, we will find ourselves using some rather awkward constructions, needed to explicitly disambiguate grammatical constructions like this last one. Please bear with us, now that you have seen why we may need to!)

The distinction between distributive and collective readings of predications can be made also for resumptive plural pronouns: '...they sang' and '...they entail Q' have the same ambiguity. Thus the quantified sentences

> There are some men such that they sang.

and

> There are some sentences such that they entail Q.

are ambiguous.

In principle, the distinction between the plural and collective readings can be made for any occurrences of plural noun phrases as subjects (or "direct objects", etc.) of predication, no matter whether the resulting readings are metaphysically bizarre or not. Sometimes it is hard to see.

> John and Paul walked.

could mean not only what it usually means, but also that John and Paul collectively performed the activity of walking. Note that this collective reading is not what we usually mean by

> John and Paul walked together.

What we usually mean by this is something in the neighborhood of:

> John walked and Paul walked, and when they (each) walked, they were knowingly and continually near one another and were knowingly engaged in some kind of interaction.

This reading does not say that John and Paul collectively performed an activity of walking; rather, it says that they each performed his own walking activity, and the two performances were co-ordinated in some fashion.

So the English 'together' does not always serve to express a purely collective reading. The collective reading for 'walk' can be brought out with a little imagination as follows: There is a very intelligent and highly social species of snake. When a group of these snakes perceives certain kinds of need, the snakes will entangle with one another forming an upside-down 'U' about three times as tall as the average snake is long. They cooperatively move in a way that can be described as walking, which allows them, for example, to get over streams none individually could cross. If there were such snakes, we could say, of some of them, that they (together) walked, while denying that any one of them walked.

It does not matter if such a species is really possible; what matters is that the verb 'walk' has a truly collective reading. As it does, even the sentence

John and Paul walked together.

has such a reading, though the reading is (usually) false.[3]

Predication of adjectives is subject to the basic ambiguity as well.

John and Paul are interesting.

could mean that each is interesting, or that they, collectively, are (but maybe neither is interesting on his own).

It is less clear that predicates involving common nouns have the same sort of ambiguity.

John and Paul is a man.

is simply not grammatical English, while

John and Paul are men.

---

[3]Since the form

PLURAL-NOUN VERB together

can express both the purely collective reading (with a single VERB-ing and collective agency) and the semi-collective reading (with multiple, co-ordinated, VERB-ings, one for each of PLURAL-NOUN), 'John and Paul lifted the piano' actually has (at least) two non-distributive readings. (Compare 'John and Paul threw the ball together.')

seems only to mean that

> John is a man and Paul is a man.

But then there is

> John and Paul are a man.

I believe that this is grammatical, and expresses a false proposition, one that is perhaps more clearly brought out by

> There is something, and it is a man, and John and Paul are (collectively) identical with it.

The adjectival phrase 'identical with it' is subject to the distributive/collective ambiguity, just as are 'heavier than it' and 'arranged in a shape similar to its'.

Some predications might seem at first to have *only* a collective reading.

> John and Paul parted.

cannot be read, one might think, as

> John parted and Paul parted.

But this is not really so. The first sentence might be elliptical for

> John and Paul parted with Mary.

which has both kinds of reading. Alternatively, it could mean that

> John and Paul parted with one another.

This, the most common meaning, turns out to be neither simply distributive (it does not say 'John and Paul each parted') nor collective: it says, in effect, that

> John parted with Paul and Paul parted with John.

The force of 'one another' is that of a more elaborate form of "distribution":

> Every one of John and Paul is such that it parted with every other one of them.

2.2   Collective vs. distributive in metaphysical logic

When we formalize English sentences, at least in introductory logic courses, we assimilate common nouns, intransitive verbs, and some adjectives to a grammatical category of "one-place predicates"; we assimilate transitive verbs and certain adjectival constructions to a grammatical category of "two-place predicates". So much is mere grammar.

When we give a model-theoretic semantics, we typically interpret n-place predicates as n-tuples formed from elements of a domain of discourse. So much is mere mathematics.

Some philosophers also take the formal grammar to mirror metaphysics in the following way: there is a category of "atomic" facts (propositions) that a given language can express, and all other facts (propositions) that it can express are "built out of" them by logical operations on them; further, each atomic fact can be seen as the bearing of a property by an object, or the bearing of an n-place relation by n objects (or perhaps n objects *in an order*).

Now suppose that this metaphysics is basically right. And suppose that many ordinary and scientific "predicates" express properties and relations, so that, for example

John lifts the piano.

expresses the proposition (and, if it is true, the fact) that John bears the relation of lifting to the piano[4] and

That cannonball is moving at rate r in direction d (relative to my reference frame).

expresses the proposition (and, if it is true, the fact) that a certain object has a certain velocity-property (or, perhaps, that it bears a certain velocity-relation to its user's reference-frame).

Thinking of English sentences and atomic facts this way,

John and Paul (collectively) lift the piano.

---

[4]It is likely that some words do not really express properties and relations, but only appear to; for convenience of exposition, I will suppress qualifications like "assuming that this predicate expresses a property".

presents a problem. Whereas the distributive reading can be thought of as expressing (when true) a conjunction of atomic facts, the collective reading must be approached differently.

One approach would be to think of 'lift' as *here* expressing a 3-place relation, that holds among John, Paul, and the piano. But then

> John, Paul, and George (collectively) lift the piano.

would involve a different relation, and

> Some men (collectively) lift the piano.

would involve covert restricted quantification over relations; it would say something like

> There are some men, and there are some relations, and each of those relations is a "lifting relation", and those men bear one of those relations to the piano.

Besides the implausibility of the suggestion of the covert quantification over relations, this approach faces the problem of clarifying the notion of a "lifting relation," which would appear to be a new category of property of relations.

Perhaps 'lifts' expresses a single "multigrade" relation.[5] A multigrade relation can apply *as if* it had any number of blank spots. But what exactly does this mean? Most attempts to make this out have focussed on formal logic and semantics, rather than metaphysics.[6]

I suggest that we think of

> John and Paul (collectively) lift the piano.

as expressing (if true) the fact that the lifting relation holds between, John and Paul (collectively, not each), on the one hand, and the piano, on the other. The lifting relation involved is the very same two-place relation that is involved in the fact that John lifts the guitar. It is a "two-place" relation, but more than one thing can (simultaneously, so to speak) fill one of its places.

---

[5]The term seems to come from Leonard and Goodman [4].

[6]As in [6] and [9].

This conception has a great advantage over conceptions on which what is really going on in this case involves a three-place relation (or three-place instance or determinate of a multigrade relation). Consider the difference between (the collective readings of)

> John and Paul fight George and Ringo.

and

> John and Paul and George fight Ringo.

If 'fight' in both examples acts as a four-place relation, it would seem that the very same proposition, one we might represent as

> *Fight*(john, paul, george, ringo)

is being expressed by both sentences. Clearly, the two English sentences are not logically equivalent. Thus it is much better to think of 'fight' as once and for all expressing a two-place relation. We would then represent the propositions expressed by the two sentences as something more like

$$\textit{Fight}(\underbrace{\text{john, paul}}, \underbrace{\text{george, ringo}})$$

and

$$\textit{Fight}(\underbrace{\text{john, paul, george}}, \underbrace{\text{ringo}})$$

respectively.

This conception generalizes neatly: we may think of every property and relation has having a fixed finite "arity", but as "accepting" any thing *or any things* in any of its "blank spots". This is not to say, of course, that you get a *fact* when you put some things in the one "blank spot" of any property, just that you get an (objectual) proposition, a "logical possibility of a fact"—at any rate, you get something of the same kind as what you get when you put single things in each of the blank spots. (By an "objectual" proposition I mean roughly what is usually meant by a "singular" proposition—a proposition that directly involves objects; for obvious reasons, that term is potentially misleading here.)

One upshot of our conception is that there is no fundamental metaphysical distinction between "plural" properties and "singular" ones.[7] There may be, of course, properties that are actually *possessed* only by single things. But the propositions formed by predicating such a property exist nonetheless—the point is that they are false. Similarly, there may be properties that are actually possessed, but never by single things. Again, the propositions formed by predicating such a property of one thing will simply be false, not non-existent.

3   The Atomist interpretation of natural language

We now return to the Extreme Atomist. The metaphysics advocated above is most congenial to the Atomist, for the Atomist may now hold, for example, that the very property of being human, the property expressed by the natural language predicate '__ is human', is never actually possessed by a single thing, but only possessed by many atoms collectively. Thus, in rough outline, the Atomist will say that in interpreting natural (ordinary and scientific) language, the only adjustment needed (from the kind of conception a non-Atomist has) is in the *number* of things that are referred to; no substantial adjustment need be made for the predicate places. (The only *illusion* that we are under, if we are under one, is that what we refer to with a grammatically singular expression is not really a single thing.)

3.1   The grammatically singular

It is fairly easy to see how the Atomist will treat grammatically singular talk. When we say, for example

John is human.

what we are really doing is attributing to some atoms, collectively, the property of being human. (A nice way to put this is: the atoms referred to by John *constitute* a man—*not* meaning that there is a man that they constitute, but that they collectively have the property of being human.)

---

[7]We may be, therefore, in disagreement with the most basic metaphysical ideas in other current approaches to plural language. Byeong Yi accepts that there is such a distinction, if I understand him right.

The basic ideas are these: grammatically singular names actually refer to many atoms; predication of a predicate that expresses a one-place property, to a name, expresses the objectual proposition that results from filling the blank spot of the property with (collectively) the atoms that the name refers to.

Note that the Atomist does not assert that

> John is human.

means

> The atoms referred to by 'John' (collectively) have the property of being human.

That is an implausible doctrine: the original sentence is not about the name 'John' at all. But the latter sentence does give the former's truth conditions.[8]

Singular quantification in natural language will be interpreted with plural quantification:

> There is a human.

has as its truth-conditions, essentially this:[9]

---

[8]As for *meaning*, the Atomist can advance the idea that the proposition expressed by

> John is a man.

is an objectual proposition that could, in principle be expressed by a sentence formed by an extremely long list of names of atoms followed by "(collectively) are human" (assuming that John is composed of finitely many atoms). The Atomist could also hold that the name is *directly referential*; it does not refer to the atoms it refers to in virtue of having some other, more primary semantic value. He could also hold that it came to pass that the name refers to the atoms it refers to exactly because of some causal link between those atoms, the name, and the linguistic community that uses the name. Alternatively, the Atomist could hold a more descriptivist theory of names, so that the name refers to its atoms in virtue of expressing some kind of descriptive property, and its being the case that those atoms are the atoms that have that property. (In most of the rest of the paper, we will ignore issues of *meaning* as such, but will confine ourselves largely to issues of truth-conditions and that aspect of meaning that is captured by properties, relations, objects, and propositions built from them with logical operations. See section 5.1 for a little more about meaning.)

[9]The exact statement would make use of an analog of the "variable assignments" of standard first-order model theory. We will give such a statement later, good for both singular and plural "variables" of natural language.

> There are some atoms such that they (collectively) are human.

The general pattern for singular predications, whether of a name or of a bound pronoun, is thus

> $\ulcorner$t is $\phi\urcorner$

is true (satisfied) just in case

> the atoms that (collectively) t refers to (collectively) satisfy ϕ

Identity in the singular case

One can begin to see the outlines of a systematic Atomist semantics for the singular fragment of natural language, in which the ontology includes nothing but atoms. Of special note are the Atomist construals of the relational expressions "is identical with" and "is part of", which we pause to consider here. Given what we have said, the Atomist will automatically (assuming that 'is identical with' expresses a two-place relation) interpret

> a is identical with b

so that it has the truth-conditions

> The atoms referred to by 'a' are (collectively) identical with (collectively) the atoms referred to by 'b'.

(This is just an application of the general pattern just given.) Given the picture of properties and relations in 2.2, and given that identity is a two-place relation, such propositions as the above truth-condition expresses should exist. That is, there is a proposition formed by filling one of the two blank spots of the identity relation with some things, collectively, and filling the other with some things, collectively.

Presumably (though it is not clear how often the question has been considered), some of the resulting propositions are true. In particular, if a proposition is formed by putting some things (collectively) in one of the blank spots, and putting *those things* in the other blank spot, the result is a true proposition.

Now, in the Atomist's language, there is a plausible general principle governing instantiations of the part-whole relation by many things collectively. It is this: for any atoms (atoms$_1$) and any atoms (atoms$_2$)

atoms$_1$ are (collectively) identical with atoms$_2$ (collectively)

is equivalent with

every one of atoms$_1$ is one of atoms$_2$ and every one of atoms$_2$ is one of atoms$_1$

(We will see later that the Atomist will hold that an analogous equivalence *fails* in natural language. This is near the heart of the logical difficulties we are trying to solve.) Thus, the above truth-condition appears to be equivalent with

Every atom that is one of the atoms referred to by 'a' is one of the atoms referred to by 'b', and vice-versa.

Parthood in the singular case

Given what we have said, the Atomist will automatically (assuming that 'is part of' expresses a two-place relation) interpret

a is part of b

to have the truth-condition

The atoms referred to by a are (collectively) part of (collectively) the atoms referred to by b.

As with identity, there is a plausible general principle governing instantiations of the part-whole relation by many things collectively. It is: for any atoms (atoms$_1$) and any atoms (atoms$_2$),

atoms$_1$ are (collectively) part of atoms$_2$ (collectively)

just in case

every one of atoms$_1$ is one of atoms$_2$

(Again, an analogous principle will fail in natural language.) Given this principle, the truth-condition above is equivalent to

Every atom referred to by a is referred to by b.

3.2   The grammatically plural

The Atomist interpretive program hits serious technical difficulties when it comes to interpreting *plural* constructions in natural language. These occur even before we face the logical problems considered at the beginning of the paper.

First, an apparent non-difficulty: on the face of it

> John and Paul each lifted the piano.

can simply be taken to mean

> John lifted the piano and Paul lifted the piano.

We have seen how each conjunct will be interpreted, and it is no problem to interpret the sentential connective 'and'.

What goes in the blank spot of a property?

Next, consider how the Atomist should construe

> John and Paul (together) lifted the piano.

Since the Atomist sees this as basically asserting the holding of a two-place relation, the problem is to find the relata, so to speak.[10] In the resulting proposition, what is in the first blank spot?

We know that

> John lifted the piano.

is to be construed as, effectively, asserting the proposition that results from filling the first blank spot of the two-place "lifted" relation with some atoms (John's atoms) (collectively) and filling the second blank spot with some other atoms (the piano's atoms) (collectively). What would it mean to put, into that first blank spot, *both* John's atoms (collectively) and Paul's atoms (collectively)?

Our answer is as follows. It is nothing more or less than to put the following atoms into the first blank spot (collectively): the atoms, call them 'atoms$_{jp}$', such that each of atoms$_{jp}$ is one of the atoms referred to by 'John'

---

[10]But only "so to speak", since what goes in a blank spot is no single *relatum*.

or one of the atoms referred to by 'Paul'; and every atom that is one of the atoms referred to by 'John' or is one of the atoms referred to by 'Paul' is one of atoms$_{jp}$.

Note well that we cannot call atoms$_{jp}$ "the atoms that 'John' and 'Paul' both refer to". In fact, there are no such atoms: 'John' refers to John's atoms (collectively), and to no others, and 'Paul' refers to Paul's atoms (collectively), and no others. Note well that we cannot call atoms$_{jp}$ "the atoms that either 'John' refers to or that 'Paul' refers to." Neither 'John' nor 'Paul' refers to atoms$_{jp}$, and in fact the definite description 'the atoms that either 'John' refers to or that 'Paul' refers to' fails to be felicitous—though there are atoms such that ('John' refers to them or 'Paul' refers to them), the uniqueness implied by 'the' does not obtain: there are some such atoms, and some other such atoms.

To make this perfectly clear, consider a more ordinary example of a similar issue, taken from McKay: suppose that in the library, there are some students meeting together, and there are some other students meeting together.[11] Let it be that students a, b, and c are meeting together in Room 101, and students d, e, and f are meeting together in Room 102, and no other students are meeting together in the library. Then the phrase 'the students who are meeting together in the library' is infelicitous because of a failure of uniqueness. Note that if we regard the description as picking out all six students, we get the unhappy result that it is true that

> The students who are meeting together in the library are not meeting together in the library.

It's better to regard the description as infelicitous. It's better to regard the displayed sentence as semantically problematic (in this circumstance), and truth-value-less or false, rather than as straightforwardly true.

Perhaps English has a means of compactly expressing the operation that corresponds to the one we used to define our term 'atoms$_{jp}$', the one we would use to generate a felicitous description that picks out the six students (and no others).[12] Perhaps 'the sum total' has this effect when

---

[11]See Thomas McKay's forthcoming *Plural Predication* for a careful discussion of the way 'the' behaves in conjunction with plural noun phrases.

[12]E.g., 'the students such that each of them is among some students meeting together in the library, and such that each student that is among some students meeting together in the library is among them'.

it replaces 'the', so that we may call atoms$_{jp}$ 'the sum total of atoms that are either John's atoms or Paul's atoms', and call the six students 'the sum total of students meeting together in the library'. If not, let us agree to use 'the sum total' this way, since we will frequently need the operation.

Our question was "What would it mean to put, into that first blank spot, *both* John's atoms (collectively) and Paul's atoms (collectively)?" Another answer might try to make use of a notion of "pluralities" of atoms. To put John's atoms and Paul's atoms into the blank spot of a relation would be to put two things, each a plurality of atoms, into the blank spot. But this seems to be nothing more than a cryptic way of expressing what we have just described more carefully. To put *one* plurality of atoms into a blank spot, (if we accept this way of speaking) is to put those atoms (the ones in the plurality) (collectively) into the blank spot. To put *two* pluralities in is the same as putting in the *one* plurality that is their sum total.

Another answer would make use of "plurally plural" talk: To put John's atoms and Paul's atoms into a blank spot would be to put two "atomses" into the blank spot. Again this might be just a cryptic way of expressing the answer we have given. If not, we need elucidation.[13] On behalf of the Atomist, we hold that plurally plural talk is not metaphysically significant—it is at best a mere verbal code for plural talk, exactly because what it is for two "atomses" to have a property is either (1) for each of them to have it (in which case we have nothing new) or (2) for the atoms, atoms$_+$, to collectively have the property, where atoms$_+$ are the atoms such that every one of atoms$_+$ is in one or the other plurality and every atom in one or the other plurality is one of atoms$_+$—the atoms that are the sum total of the two "atomses" we started with. One can certainly make sense of a term t, an allegedly plurally plural term, that bears a relation (call it 'reference') to some atoms (collectively) and some other atoms (collectively) and maybe still other atoms. (In fact, that is just how we will think of the grammatically plural terms of natural language!) But there are no metaphysically basic facts expressible with plurally plural terms that are not expressible with plurals, though there are many metaphysically basic facts expressible with plural terms that are not expressible with

---

[13] Rayo, in [?], defends the notion of the plurally plural.

singular terms.[14]

Returning to our question, and our answer, we have that

John and Paul (together) lifted the piano.

effectively puts the atoms$_{jp}$ in the first blank spot of the relation of lifting, and puts the piano's atoms in the second blank spot. It might seem natural, then, to regard 'John and Paul' as referring to atoms$_{jp}$. But consider

John and Paul (each) are human and (together) lifted the piano.

If 'John and Paul' simply refers to atoms$_{jp}$, then how are we to connect them up to the property of being human in the right way, while also connecting them up to the property of lifting the piano, in another way?

A quick solution is to distribute first, then run the semantics. We first transform the sentence into

John is human and Paul is human and John and Paul together lifted the piano.

and then give the expected semantics of this. This avoids the problem, but a similar move is simply unavailable for

There are some things such that: each of them is human, and they together lifted the piano.

Given that we have here, in effect, two occurrences of the same resumptive pronoun, we face the semantic problem we avoided above. But since we have only the pronoun, there simply is no analogous distributed form.

The key to overcoming the difficulties

The problem now is to fix on a single type of semantic valuation for a plural term, whether grammatically complex (like 'John and Paul') or grammatically singular (like 'them') that carries enough information for both distributive and collective predications.

---

[14]There may be "quantificational" facts expressible with the plurally plural, however, that cannot be expressed with plurals and without quantification over properties. See 4.5 below. But we see sentences that express such facts as verbal code for sentences without plurally plural expressions, but with quantifiers over properties and relations. See footnote 15 for further discussion of the hierarchy: singular, plural, plurally plural ...

The solution is to refine our conception of the way natural language terms (constants and variables) can refer to atoms.

We will take advantage of the fact that there are two different dimensions of multiplicity, as it were, when a single thing (such as an expression) is connected to many things. The connection can be distributively multiple, and it can be collectively multiple. Thus, an expression might *refer* multiply in either or both of these ways. It might refer to α and refer to β and refer to γ (distributive); it might refer to α, β, and γ (collectively); and it might refer to α and β (collectively) and refer to γ, and refer to nothing else. Note that if this last possibility is realized, the term does not refer to α, nor to β.

There are other relations like this. Let α be the symbolic sentence $(S \rightarrow \neg Q) \wedge (P \rightarrow Q)$, let β be $(S \rightarrow \neg Q) \wedge (\neg P \rightarrow Q)$ and let γ be $S \wedge Q$. Consider the relation R that holds between a sentence ϕ and some sentences just when (*i*) those sentences (collectively) entail ϕ, and (*ii*) those sentences do not entail a contradiction. Then Q bears R to α and β (collectively), Q bears R to γ, and Q bears R to no other sentences (that are such that each of them is among α, β, and γ).

The general strategy will be to take the grammatically singular terms of natural language to refer plurally, but *only collectively*; thus, if a singular term refers to some atoms "atoms$_1$" (together), then they are the only such atoms—any atoms it refers to are (collectively) identical with (collectively) atoms$_1$ (which is equivalent with its being the case that for any atoms$_2$ that it refers to, each of atoms$_1$ is one of atoms$_2$, and vice-versa). Another way to put it: grammatically singular terms refer only once. Grammatically plural expressions, by contrast, will refer multiple times, as well as collectively plurally. That is to say, there may be some atoms that a plural term refers to (collectively), and also some other atoms that it refers to (collectively), and perhaps still others that it refers to, and so forth. It could refer arbitrarily many times, each time to arbitrarily many atoms (collectively).

For example,

John and Paul

will be treated as follows. 'John' refers to some atoms (collectively) (atoms$_j$) and to no other atoms. 'Paul' refers to some atoms (collectively) (atoms$_p$) and to no other atoms. 'John and Paul' refers to atoms$_j$, and also refers to atoms$_p$, and it refers to no other atoms. In general, a plural term that is a list of other terms will refer to any atoms (collectively) that some term in

the list refers to (collectively) and to no others. Note that 'John and Paul' does *not* refer to atoms$_{jp}$, the sum total of atoms$_j$ and atoms$_p$. (Again, to be exact, atoms$_{jp}$ are those atoms such that: each atom that is either one of atoms$_j$ or one of atoms$_p$ is one of atoms$_{jp}$; and every atom that is either one of atoms$_j$ or one of atoms$_p$ is one of atoms$_{jp}$.)

To give truth-conditions for

> John and Paul (each) are human and (together) lifted the piano.

we then need to give truth-conditions for each conjunct

> John and Paul (each) are human

and

> John and Paul (together) lifted the piano

that play off of the referential facts about 'John and Paul'.

The truth-conditions for the first conjunct will be

> For any atoms, if those atoms are referred to by 'John and Paul', then they (collectively) have the property of being human

This is an instance of the general idea that for any plural term $\ulcorner t \urcorner$ (whether a grammatically complex list or a grammatically simple bound pronoun)

> each of t is ϕ

will be true just in case

> for any atoms, if they are referred to by $\ulcorner t \urcorner$, then ϕ is true of (satisfied by) them (collectively).

The truth conditions for the second conjunct,

> John and Paul (together) lifted the piano

are

> The sum total of atoms referred to by 'John and Paul' (collectively) have the property of lifting the piano. Given our definition above, these are the atoms atoms$_{jp}$.

This is an instance of the general idea that for any plural term $\ulcorner t \urcorner$ (whether a grammatically complex list or a grammatically simple bound pronoun)

> $\ulcorner$t (collectively) are $\phi \urcorner$

will be true just in case

> the sum total of atoms referred to by t are such that ϕ is true of (satisfied by) them (collectively)

Reference and reference[*]

The connection between a term $\ulcorner t \urcorner$ and the sum total of atoms referred to by it will be needed frequently, so we will introduce some new vocabulary for it.

Let us say that a plural term *refers** to (distributively) those atoms atoms$_*$ that are such that: any atom that is one of some atoms that it refers to (collectively) is one of atoms$_*$, and any atom that is one of atoms$_*$ is one of some atoms that it refers to (collectively); it *refers** to nothing else. Thus if a plural term $\ulcorner t \urcorner$ refers to α and β (collectively), and also refers to γ and δ (collectively), and refers to no other atoms, then t refers* to α, refers* to β, refers* to γ and refers* to δ, and refers* to nothing else.

Thus, the above truth-condition for collective predications may be expressed as

> the atoms that $\ulcorner t \urcorner$ refers* to (collectively) are such that ϕ is true of (satisfied by) them (collectively)

The reason we do not simply take a plural term to refer to what it refers* to is that this would collapse the articulation of the world associated with the term as a result of its multiple reference, a semantic feature of it that is picked up on in the contexts of expressions like 'is one of' and 'are among'. Consider again the interpretive pattern we gave above for

> each of t is ϕ

roughly put,

> Any atoms (collectively) that t refers to satisfy ϕ.

Quantifiers and pronouns

Our treatment of the reference of plural terms can now be extended to an exact treatment of plural pronouns bound by plural quantifiers, in which we treat them as "referring" relative to an *assignment relation*. (Some readers may want to skip our discussion of this somewhat technical issue, and go ahead to section 4.)

> There are some things such that: each of them is human, and they together lifted the piano.

will be treated so that its truth conditions are roughly as follows:

> It is logically possible for there to be a term t that refers both collectively and distributively plurally and such that: "each of t is human, and t (together) lifted the piano" would be true.

Our semantic treatment of 'John and Paul are each human and together lifted the piano' played off the semantic facts about 'John and Paul' *without looking inside, syntactically, that noun-phrase.* That is why there is no problem about extending it to a treatment of arbitrary, including syntactically atomic, noun-phrases of the same *semantic* category—in particular, of bound plural pronouns.

This talk of logical possibility is a rough expression of an idea that should be compared with the following as a statement of the truth conditions for an existential sentence $\exists x \phi(x)$ in a traditional setting:

> It is logically possible for there to be a term t such that $\phi(t)$ would be true.

When it comes to giving a rigorous statement of the truth conditions for a quantified wff, we replace the logical modality of possibility with the existence of an actual function (called a variable assignment) which behaves in an appropriate way. The space of all "logically possible referring terms" is realized as the space of functions from variables to objects in the domain.

To effect the same sort of realization, we cannot use traditional functions, nor even traditional set-theoretic relations; instead we must use relations that relate a variable multiply, in both ways (with both collective and distributive multiplicity). We could *simulate* such relations with traditional set-theoretic relations that relate a term to some (possibly more

than one) subsets of the domain; we would regard relation to a subset as simulating collective relation to the members of the subset. But it is much more in the spirit of the current approach to use the real thing instead of a surrogate: we will use unanalyzed relations that relate multiply in both ways.

Assume for the moment that we have a sensible notion of singular and plural "variables" corresponding to the bound singular and plural pronouns in English singular and plural quantificational statements. We will say that a relation R is an "assignment relation" if (1) it relates each singular variable exactly once—that is, relates it to some atom or to some atoms collectively, and relates it to no other things; and (2) relates each plural variable at least once—that is, relates it to an atom or to some atoms collectively, but may relate it also to another atom or to some other atoms collectively. Now we can realize the possibility in

It is logically possible for there to be a term t that refers both collectively and distributively plurally and such that: "each of t is human, and t (together) lifted the piano" would be true.

as

There is an assignment relation R such that (1) for any atoms R relates 'they' to, those atoms are (collectively) human; and (2) the atoms, call them 'atoms$_1$' that are the sum total of atoms R relates 'they' to (collectively), lifted the piano. To be exact, atoms$_1$ are those atoms such that: each atom that is one of some atoms R relates 'they' to is one of atoms$_1$; and every atom that is one of some atoms R relates 'they' to is one of atoms$_1$.

The condition for the second conjunct is, of course, just like that for 'John and Paul together lifted the piano', except that instead of reference, we talk of what atoms are assigned by R to a pronoun. If we run together reference and value under an assignment, the condition becomes

the atoms that (collectively) 'they' refers* to (relative to the assignment relation R) (collectively) lifted the piano

Plenitude of properties

To ensure that there are enough assignment relations to do the needed work, it suffices to assume a few principles of plenitude about properties

and relations of atoms. We will quantify, plurally, even, over such properties and relations. We make no attempt to analyze such quantification, but do wish to emphasize that *we do not predicate anything of the properties and relations we quantify over*. (We assume that to say, for example, that there is a property had by John, does not involve the notion of predicating anything of a property, but rather involves the notion of predicating a property, of John. This is very different from saying, for example, that there is a property *that is eternal*, or there is a property *P* had by John and a property *Q* had by John *and P is not identical with Q*.)

Here are our principles. The first two are analogous to what, in a classical setting, we might put roughly as "there is a property for any arbitrary extension (set of things)". The second two link properties to relations in such a way as to guarantee that there will be analogs of the classical notion of an arbitrary variable-assignment.

1. For any atoms, there is a property had by them collectively, and had by nothing else.

2. For any properties, there is a property that is had by an atom or some atoms (collectively) if and only if it or they have one of those properties.

3. For any things$_1$ and any property, there is a (two-place) relation such that (A) things$_1$ (collectively) bear it to some things$_2$ (collectively) just in case they$_2$ (collectively) have the property; and (B) the relation relates no other things.

4. For any (two-place) relations, there is a (two-place) relation that relates some things (collectively) to some things (collectively) just in one of those relations does.

Once we have dotted all the technical 'i's and crossed the 't's, we will have a completely systematic interpretation of a significant portion of the natural language.

4   Results

Before we continue to explore the ramifications of the Atomist semantics, let us work through some elementary considerations that will be useful in

the sequel and will help to solidify one's grasp of some key elements of the semantics.

A plural term may refer many times, each time to an atom or some atoms collectively, while a singular term refers only once.

(In this formulation, we harmlessly run together, under the name "reference", (1) the notion of *reference* for a name or term built out of names, with (2) the notion of being related to a variable under an assignment relation; a practice we will continue.)

Recall that we say that a plural term *refers** to each atom that is one of the sum total of atoms it refers to—each one of those atoms atoms$_*$ that are such that: any atom that is one of some atoms that it refers to (collectively) is one of atoms$_*$, and any atom that is one of atoms$_*$ is one of some atoms that it refers to (collectively). Consider again a plural term $\ulcorner t \urcorner$ that refers to α and β (collectively), and also refers to γ and δ (collectively). It refers* to each of α, β, γ and δ (collectively), and to no other atoms.

Notice that two plural terms $\ulcorner t \urcorner$ and $\ulcorner s \urcorner$ may refer* to the same atoms (collectively) while (1) there are *no* atoms that are (collectively) referred to both by t and by s; or (2) there are *some* atoms that t refers to (collectively) that s also refers to (collectively) and some that they do not both refer to collectively. For example, if t is as in the example above, and s refers to α, refers to β, refers to γ, and refers to δ, and to nothing else, condition (1) is met. And if s refers to α and β (collectively), and refers to α and γ (collectively), and refers to α and δ, (collectively), then condition (2) is met.

Notice that if t and s refer* to the same atoms, but either condition (1) or (2) above is met, then

> t (collectively) are identical with s (collectively)

will be true, while

> every one of t is one of s and every one of s is one of t

should be false, since there are some atoms that one of the two terms refers to that the other does not. This will be born out in detail below, but for now it will do to say that the Atomist truth condition for

> every one of t is one of s

will be equivalent with

Any atoms referred to (collectively) by t are referred to (collectively) by s

It is useful to extend the notion of reference* to singular terms as well: let us say that a singular term refers* to *each of* the atoms that it refers to (collectively). In the case of singular terms, reference* "distributes" reference. In the case of plural terms, reference* might aptly be called the "un-articulation" of reference, or the "total distribution" of reference.

4.1   Analogies and disanalogies with sense/reference

There is an analogy between, on the one hand, our notions of reference and reference*, and, on the other, Frege's notions of sense and reference. For Frege, sense determines reference, and not vice-versa. For the Atomist, reference determines reference*, and not vice-versa. For Frege, sense presents reference. For the Atomist, reference articulates reference*. In the case of 'John and Paul' the many atoms it refers* to, each of atoms$_{jp}$, are "articulated" into exactly two vast ensembles, as it were, and it would not be inappropriate to say that this articulation *presents* the atoms$_{jp}$ in a certain way. (Consider how differently they are "presented" by the term 'atoms$_{jp}$' which presents them one by one, so to speak.) Similarly, 'John' presents John's atoms as one. For Frege, sense is a crucial element of the cognitive significance of a sentence. No doubt the different articulations associated with 'John' and 'John's atoms', for example, have a certain cognitive significance for the users of the words.

For Frege, for a sentence of the form

⌜t is F⌝

with F a "normal" predicate, it is the reference of t, determined by its sense, that most directly relates to the truth-conditions; if s is a term with a different sense, but the same reference,

⌜s is F⌝

certainly has the same truth-value. For the atomist, for a sentence of the form

⌜t are collectively F⌝

with F a "normal" predicate, it is the reference* of t, determined by its reference, that most directly relates to the truth-conditions; if s is is a term with a different reference, but the same reference*

⌜s are collectively F⌝

certainly has the same truth-value.

For Frege, in certain contexts, it is the sense of a term that ends up, in that context, making the kind of contribution to truth-conditions that its reference makes in the more bare or normal contexts. In these contexts, terms with the same reference may *not* be inter-substitutable *salva veritate* (though they will be if they have the same sense). For the Atomist, in certain contexts, it is the reference of a term that is most directly relevant to the truth-conditions, rather than reference*. Consider, for example, the context ⌜John is one of ___⌝. Inserting 'John and Paul' into the blank and inserting 'atoms$_{jp}$' yield different truth-values, even though the terms have the same reference*. But inserting terms with the same reference is guaranteed to yield the same truth-value. (We will discuss these contexts, and their difference from "normal" contexts further, in section 4.5.)

But there are disanalogies: reference and reference* are metaphysically on the same level (as opposed to being in The Third Realm and The First Realm, respectively); in the case of some terms, reference and reference* are exactly the same (e.g. 'atoms$_{jp}$', 'John's atoms', 'this atom'); in the case of singular terms (but not plural terms), reference* determines reference (as well as vice-versa).

What is more, and crucial, the Atomist (the one I am presenting, anyway) is not Fregean about propositions: we do not take it that if two terms t and s have different reference, then they must make different contributions to the propositions expressed by the sentence in which they are used:

John is talking.

and

John's atoms are (collectively) talking.

express exactly the same objectual proposition: the proposition that results from putting certain atoms into the blank spot of the property of talking. Any differences between these sentences at the level of *cognitive significance* are beyond the scope of this paper. And, unlike Frege, we do not

hold that there is a semantically significant distinction for predicates, or for whole sentences, that is like the distinction we are making for referring terms (between) reference and reference*. By and large, we are much more Russellian than Fregean. And, unlike Frege, we do not see in natural language any mechanism that could generate arbitrarily high new levels of the same kind (like iterating 'believes that' may have been for Frege).[15]

4.2   Composition

As noted at the outset, the Atomist semantics for natural language validates the natural language's expression of the view that Composition is Identity.

We are now in a position to explore this facet of the semantics in greater detail. The Atomist truth-conditions for a clause of the form

> s is identical with t (collectively)

with singular term ⌜s⌝ and plural term ⌜t⌝, says that this is true just in case

> the atoms that s refers to are (collectively) identical with (collectively) the atoms t refers* to

If ⌜s⌝ is also a plural term, the clause for

> s are (collectively) identical with (collectively) t

is

---

[15] But cf. footnote 13. If it made sense, as Rayo and others suggest, that there be an infinite hierarchy that goes (singular, plural, plurally plural, . . .) and if natural language had terms of all these kinds then there would appear to be a hierarchy like this.

Even with our resources, we could (with some technical awkwardness) introduce another level—the level of the plurally plural, with an attendant notion of super-reference. (A term would be associated with "atomseses" not in the way that our plural terms might fancifully be said to refer to "atomses". Our plural terms bear a relation (reference) to some atoms, and maybe some other atoms, and so on. The plurally plural term would be associated semantically—somehow—with some "reference relations", and would super-refer to one atomseses in virtue of bearing one of its reference relations to each and every one of the atomses among it. But (1) even then, there are structural differences from the Fregean hierarchy; and (2) the hierarchy seems to stop there, since we deny the intelligibility of plurally plural quantification over relations, in our meta-language.

the atoms that s refers* to are (collectively) identical with (collectively) the atoms t refers* to

We can now see that

There is something such that John and Paul are (together) identical with it.

will be true. As we did above, let atoms$_j$ be the atoms such that 'John' refers to them (collectively) (and to no other atoms), let atoms$_p$ be the atoms that 'Paul' refers to, and let atoms$_{jp}$ be the atoms that result from "putting together" atoms$_j$ and atoms$_p$. The expression 'John and Paul' refers to atoms$_j$ (collectively) and atoms$_p$ (collectively), and refers to no other atoms; it refers* to each of atoms$_{jp}$, and refers* to no other atoms.

Since there is a variable assignment R that relates 'it' to atoms$_{jp}$ (collectively) (and relates 'it' to no other atoms)

John and Paul are (together) identical with it

is satisfied on R just in case atoms$_{jp}$ are the atoms that 'John and Paul' refers* to, which they are. Hence, the sentence will be true.

The reasoning we just considered can be generalized to show that the following is guaranteed to be true:

For any things, there is exactly one thing such that they (together) are identical with it.

Thus we may introduce (if it is not already in the natural language) a term operator for *fusions* or "mereological sums" with the following semantics:

For any plural (or singular) term ⌜t⌝, (e.g., a list of singular or plural names, or a variable relative to an assignment) let ⌜the fusion of t⌝ refer to (collectively) the atoms that t refers* to.

The Atomist's interpretation of ⌜t is part of s⌝ for singular terms ⌜t⌝ and ⌜s⌝ is

the atoms referred to by t are among the atoms referred to by s

or, equivalently,

every atom t refers* to, s also refers * to

The Atomist's interpretation of ⌜t is composed of s⌝ for singular term ⌜t⌝ and plural term ⌜s⌝ is the same as that for ⌜ t is identical with s (collectively)⌝:

> each of the atoms that are collectively referred to by t is referred*
> to by s

This can also be expressed as:

> the atoms that are (collectively) referred to by t are (collectively) identical with (collectively) the atoms referred* to by s

or as

> the atoms that are (collectively) referred to by t are (collectively) identical with (collectively) the sum total of atoms referred to by s

Composition is identity, so to speak.[16]

The semantics for 'fusion of', 'part of' and 'composition' guarantees that all of the sentences of the natural language that formulate the axiomatic principles of Classical Atomistic Mereology (CAM) are true.[17] Thus, the Atomist semantic treatment *validates* CAM—shows how those sentences are semantically (logically) guaranteed to be true.[18]

---

[16]Versions of this doctrine have been defended by Donald Baxter, in [2] and [1], and by David Lewis, in [5]. Baxter takes the view very seriously, while Lewis holds a very weak version of it. On Baxter's view, whether $x = y$ is relative to a something he calls a *count*. (Roughly, on one way of counting, x and y count as the same thing, while on another, they do not.) Lewis' version is so weak that Lewis himself makes clear that it only says that composition is *analogous* to identity, and Lewis explicitly clearly distances himself from a position like Baxter's. Our own view is different from either.

[17]Here is one way to formulate CAM, using plural quantification. (This formulation traces back to Tarski [8].) Say that everything is part of itself. Say that something is an 'atom' if it has only itself as a part. Say that two things 'overlap' if they have a common part. Say that a thing x is the 'fusion of' some things if: every one of them is part of x and every part of x overlaps one of them. We can now state the axioms as:
(1) The part-relation is transitive;
(2) For any things, there is a unique fusion of them;
(3) Everything has at least one atom as part.
It is a worthwhile exercise to confirm that (a) these axioms are indeed validated by the semantics, and (b) 'fusion' in the sense defined here in the footnote works out to be the same as 'fusion' as given semantically, above in the main text.

[18]Since one can encode arithmetic within a sentence of the plural language in which

4.3   'is part of' vs. 'is one of'

The Atomist interpretation of $\ulcorner$x is one of y$\urcorner$ for singular term $\ulcorner$x$\urcorner$ and plural (or singular) term $\ulcorner$y$\urcorner$ makes the expression importantly sensitive to the way the term $\ulcorner$y$\urcorner$ refers. The clause is:

> the atoms (collectively) referred to by x are (collectively) referred to by y

The generalization for $\ulcorner$x are among y $\urcorner$, where x and y are both plural terms is

> any atoms (collectively) referred to by x are (collectively) referred to by y

It is not possible to cast either of these conditions in terms of reference*, again because reference* collapses the articulation associated with reference.

It is possible to cast many other phrases in terms 'is one of' and quantification.[19] For example,

> Each of them is ϕ

can be rendered

> For every thing, if it is one of them, then it is ϕ

We will assume that our clauses for 'is one of' and 'are among' suffice to give us interpretations for all the basic elements of the full apparatus of plural quantification in natural language.

---

our version of CAM is stated (using only the predicate 'is part of') the set of validities of that language cannot be axiomatized.

[19]To be persnickety, 'is one of' itself contains the quantifier 'one'. To say that John is one of them seems to be to say "There is something, it is of them, and it is identical with John." But for some reason this is not good English. Transliterations of it in some other languages, e.g., Russian, are fine.

4.4 A failure of substitutivity of identicals?

We are now in a position to diagnose an argument against the very coherence of Composition as Identity. It is an adaptation of an argument from Byeong Yi, aimed against the proponent of the view that Composition is Identity.[20]

(0)	Some things compose something only if they are (collectively) identical with it.	Suppose for *reductio*
(1)	x is composed of two proper parts, y and z.	Premise
(2)	$y \neq x$ and $z \neq x$	From (1), def. of proper part.
(3)	Every one of y and z is identical with y or identical with z.	Logical truth
(4)	Every one of y and z is non-identical with x.	From (2) and (3)
(5)	x is identical with y and z (collectively).	From (0) and (1)
(6)	x is one of x.	(logical truth)
(7)	x is one of y and z.	From (5) and (6)
(8)	$x \neq x$	From (4) and (7)

Lines (0), (3), and (6) are guaranteed to be true by the Atomist semantics. The premises are acceptable. But we can now see just where the reasoning goes wrong: the truth of (5) does not (with the logical truth (6)) guarantee the truth of (7). The expression 'y and z' does not have the same reference as the expression 'x': in particular, 'x' does not refer to any atoms

---

[20]See [11]. Yi's target was not the Atomist *per se*, but given that, at least on the surface of natural language, the Atomist upholds the view that Composition is Identity, the Atomist is a natural target for arguments like Yi's.

The seed of Yi's point can be seen in David Lewis' discussion of Composition as Identity in [5]. Arguments of a similar nature can be found in a paper by Ted Sider [7], there directed against the compatibility of the doctrine of Composition as Identity with a natural treatment of plural quantification. The present paper arose in part out of considering how Sider's argument might be evaded. The argument will show, schematically, that it is impossible that there be something composed of some proper parts.

(collectively) that 'y and z' refers to—hence (7) is false. But since these expressions *refer*[*] to the same atoms, (5) is true.

Here is a model to help make things perfectly clear. Let 'x' refer to the atoms α, β, and γ collectively (and to nothing else). Let 'y' refer to α and β collectively, and let 'z' refer to γ collectively.

All of the rest follows by the principles of the Atomist semantics, with the result that (1)–(6) are true, while (7) and (8) are false. First, note that 'y and z' refers to α and β collectively, and refers to γ. Hence it refers[*] to α, β, and γ collectively and nothing else. Hence[21] (1) and (5) will be true. (2) is clearly true, given the references of the terms involved. (3) can be seen to be true without even knowing what 'y' and 'z' refer to, but putting together the interpretive clauses for 'one of' and for the list connective 'and'. (4) is true since

> ζ is one of y and z

is satisfied only when ζ is assigned to some atoms that 'y and z' refers to. And since the atoms that 'x' refers to are not referred to by 'y and z', (7) is false.[22]

4.5   Leibniz' Law and 'is one of'

We have a failure of the rough general rule that if two terms can flank an identity symbol to yield a true sentence, then they can be substituted *salva veritate* in other sentences. Of course, the rule is widely thought to fail when the occurrences of the terms in the other sentences are within the scope of epistemic operators like 'believes that'. But the failure of substitution seen above is not at all of this sort, nor does it involve any other

---

[21] Actually, we have not said anything about 'two', but all it means in this context is that $y \neq z$.

[22] Note that we can and should admit the truth of

> x is one of: something identical with y and z (collectively)

It is just that this is not to be confused with

> x is one of y and z

which is equivalent with (since x, y, and z are singular terms)

> x is identical with y or x is identical with z

familiar "opaque context". Nor do we hold that '__ is one of __' is a context that is sensitive to more than the reference of the terms it connects. In fact, it is more like this: the context '__ (collectively) are identical with __ (collectively)' is sensitive to only one aspect of the reference of the terms it connects—reference*.

But this is not at all to say that, for example,

> x is identical with y and z (collectively)

expresses anything more or less than an identity proposition. It expresses exactly the identity proposition that it should: the one that you get when you put "what $\ulcorner x \urcorner$ refers to" into the first blank spot of the identity relation and put "what $\ulcorner y$ and $z \urcorner$ refers to" in the other. In our example, the plural noun-phrase refers to α and β collectively, and refers to γ. What other atoms could it possibly be appropriate to put into the second blank spot of the identity relation than α, β, and γ (together)?

One might object that we should instead put two "atomses" (the first being α and β together, and the second being γ) into the second blank spot. We reply that this request, if it makes any sense at all, is carried out in no other way than by putting α, β, and γ (together) into the blank spot. (Cf. footnote 13.)

One might object that we should instead hold that the identity relation is "multigrade" in the sense that there is a version of it that has two blank spots and a version of it that has three blank spots, and that

> x is identical with y and z (collectively)

should be understood as loading α, β, and γ (collectively) into the first blank spot, α and β (collectively) into the second, and γ into the third. And incredulous stare is called for. Better would be a version with six blank spots, but then the proposition would be true. Better still would be to deny that more than one thing can (collectively) go into the blank spots of the identity relation to form a proposition. But if any property can accept many things (collectively) in its blank spot to form a proposition, as the Atomist ought to hold, then it is hard to see why identity would be any different.

So

> x is identical with y and z (collectively)

expresses a true identity proposition, even though the terms 'x' and 'y and z' are not interchangeable in some contexts. Note, however, that for any context of the form

>   __ is ϕ

that expresses a property,

> x is ϕ

and

> y and z are (collectively) ϕ

are guaranteed to have the same truth-value (since 'x' and 'y and z' refer* to the same atoms, and since collective predication is sensitive only to reference*). If any atoms atoms$_1$, are the same atoms as some atoms atoms$_2$, then any property had by atoms$_1$ (collectively) is had by atoms$_2$ (collectively). *There is no failure of the law that identicals have the same properties.*

What do and do not express properties

The context 'x is one of __' does not express a property, and '__ is one of __' does not express a relation.[23] When we look at the semantics for the first, we do not find appeal to a property that might hold of some atoms collectively. Rather, we find appeal to information about the reference of what goes in the blank, information that is finer-grained than can be captured by just looking at the sum total of atoms involved (that's reference*). But it is only the sum total that could (collectively) go into the blank spot of the property. The referential articulation of the sum total is lost if it goes into the spot of a property, but not if the semantics makes some other kind of use of it.

    The denial that 'is one of' in natural language expresses a relation is at the core of the Atomist's disagreement with appearance. Or, to put it from the Atomist's point of view, that there is no relation for 'is one of' to pick up on is a necessary side-effect of its role in the mechanism that generates

---

[23]There is a possible Atomist position that could accept that 'is one of' expresses a relation. This relation, however, would have to be "multigrade" in a sense that we rejected in section 2.2. I find our position, on which there are no such relations, much stronger, and am skeptical of any other metaphysical conception of the multigrade.

the illusion. The eyes see as one what is (or are), in fact, many; they see, as many, many of these pseudo-ones, when what is really there are just many atoms. But the articulation associated with reference is an objective, non-illusory reflection of the illusory ontology. The pseudo-one named by 'John' is not a real single thing, but the oneness of the connection between 'John' and the world is real; the pseudo-two named by 'John and Paul' are not *two* real things, but the two-ness of the connection between 'John and Paul' and the world is real. Those (real, not illusory) numerical aspects of reference are essential to what 'is one of' is sensitive to; hence part of what 'is one of' tracks is something that is not purely a feature of the world talked about, but has to do with the mode of talking.

This less-than-purely-worldly-ness of the objective facts expressed using 'is one of' should not be misunderstood.

It is not that there is no purely worldly fact associated with the truth of an 'is one of' statement.

> John is one of the Northwood Critics.

is associated with, in some non-trivial, but hard-to-categorize way, the disjunction of objectual propositions associated with

> John is identical with x
> John is identical with y
> $\vdots$

replacing x, y, and so on, with grammatically singular terms that refer to (the atoms of) each of the Northwood Critics. Further, there *is* a property P that corresponds to '__ is one of the Northwood Critics'. So the statement is also associated with the basic worldly proposition that John has P. (Let P be the property of being some atoms that 'the Northwood Critics' refers to, or, if this is not the same property, the identity property that is co-extensive with it—the property expressible, in the finite case, as "being identical with these atoms, or with those atoms, or with…".) Thus, even if there are infinitely many Northwood Critics, there is a purely worldly proposition associated with the original sentence.

4.6   Strange results?

Because the Atomist regards the appearance of the collective bearing of a property by some composite objects as really the collective bearing of

that property by the sum total of atoms that make up those objects, some somewhat strange results follow.

From the Atomist's point of view, for two men to sing together is for the atoms, that are all and only the atoms that are part of one or the other man, to collectively sing. This means that for any composite things, if the sum total of atoms that are parts of any of those things are the same as the atoms that make up the men, then those composite things sing. For example, if John is composed of two half-men, Johnleft and Johnright, and Paul is composed of two half-men, Paulleft and Paulright, and John and Paul sing together, then Johnleft, Johnright, Paulleft, and Paulright sing together.[24]

This result is strange. But it is difficult to come up with an example of something *obviously* false that follows from the Sub-realist's treatment. As Sider suggests in [7] about this kind of example, the Atomist may even regard these oddities as "metaphysical discoveries".

The two steps of interpretation

Reflection on these facts does reveal, however, that there is a range of predicates that appear to express properties, but do not. They are the predicates the explication of which essentially involves 'is one of'.

Consider the property, if there is one, of "being some children".[25] This would be a property that some things have just in case *each of them is a child*. According to the Atomist, there can be no such property, for the sum total

---

[24]Alternatively, the Atomist could regard 'they sing together' as a stylistic variant of 'there is an event of singing that each of them partakes in'; then we would not have the result that some atoms "sing together". See below.

[25]This is close to Yi's notion of the "plural expansion" of a "singular" property. (See **??** section 6, and elsewhere.) Yi holds that there is a distinction between singular and plural properties, corresponding to the distinction between predicates that are somehow grammatically marked as singular and ones that are similarly marked as plural. We rejected this metaphysics in section 2.2. But the notion of the plural expansion of a grammatically singular predicate is well-defined, independently of the metaphysics. (If ϕ is such a predicate, then the expansion is basically ⌜are such that each of them is ϕ⌝.) The non-Atomist could agree with us about the metaphysics, but join Yi in accepting that there is a property corresponding to the plural expansion of any grammatically singular predicate that expresses a property, and hence a corresponding operation on properties; but the Atomist cannot.

of atoms that compose some children collectively have any property that the children collectively have.[26] Hence, if some children had the property, there would be some atoms that had it, hence they would be some children, hence there would be some atoms each of which is a child. Thus the Atomist will not think of

> John and Paul are children.

as predicating a property of John and Paul collectively, but rather as a "stylistic variant"[27] of

> Each of John and Paul is a child.

In general, there are two stages in the Atomist interpretation of a sentence of natural language: first, transform the sentence into a stylistic variant in which every "hidden" 'is one of' is brought out explicitly, so that all other remaining predicates do express properties and relations. (E.g. any occurrence of 'they are children' becomes an occurrence of 'every one of them is a child'.) Second, apply the semantics to the result.

Thus far, the Atomist has supplied no algorithm for the first stage (though he has for the second). There may be an unavoidable element of linguistic insight here—just as there is in seeing which natural language sentences correspond to the same first-order symbolization. But there are regular patterns: e.g., plural common nouns seem to follow the analysis of 'children' above.

---

[26]The Atomist does believe in a property of "composing some children". This is a property had by some atoms just in case there are at least two children that they compose—just in case there are some atoms among them that collectively have the property of being a child, and some other (non-overlapping?) atoms among them that have that property. But this property does not correspond to the natural language 'are children', since

> John and Paul are children.

entails

> John is a child.

and this is not entailed by 'John and Paul compose some children'.

[27]In the sense of Kalish and Montague [**?**].

Some tricky examples

Gabriel Uzquiano discusses various difficulties that an Atomist faces in interpreting plural constructions in natural language, many of which we have solved or given the resources for easily solving.[28] First, consider the Geach-Kaplan sentence:

> Some critics admire only one another.

On the face of it, this may be trouble for the Atomist: if we take 'admire only one another' to express a one-place property, then if the plural term 'they', under an assignment, makes 'they admire only one another' satisfied, then there will be another assignment that will assign each of many atoms to 'they' that will also satisfy it. Hence, if the Geach-Kaplan sentence is true, then

> Some atoms admire only one another.

will also be true. This is unacceptable, since no atom is capable of admiration.[29]

But the Atomist has no real difficulty here, since the Geach-Kaplan sentence should not be analyzed this way, but rather as

> There are some things such that: every one of them (1) is a critic, and (2) is such that if it admires something, that thing is one of them.

When the quantifications and 'one of' constructions are given the Atomist's treatment, all is well.

Another case Uzquiano discusses requires even more transformation. Consider

> Some bricks are touching each other.

Again, there is trouble if we take 'are touching each other' as a property. The atoms that compose some bricks are not touching one another, or, may not be, as far as logic goes. Instead we must regard this as a stylistic variant of either

---

[28]In [10]. Uzquiano's explicit target is the *compositional nihilist* who wants to paraphrase sentences of natural language.

[29]See also Sider's discussion of the difficulties sentences like this present for the advocate of Composition as Identity, in [7].

> Some bricks are such that: each one of them is touching each other one of them.

or

> Some bricks are such that: each one of them is *related by the ancestral of the touching relation to* each other one of them.

The Atomist can handle the first of these straightforwardly. To handle the second, the Atomist should recast the italicized phrase, so as clearly not to have to appeal to a two-place relation. (Again, we would get unacceptable results about the atoms that compose the bricks.) We can find a rather lengthy stylistic variant, built up from these definitions:

> Say that some things are *brick-t-closed* when, every one of them is a brick, and is such that if it touches a brick, then the touched brick is also one of them.
> Say that some things x and y are *brick-r-touching* when, for any things that are brick-t-closed, if x is one of them, so is y.

The expressions 'brick-t-closed' and 'brick-r-touching' do not express properties or relations. Rather, the Atomist sees (the second reading of)

> Some bricks are touching each other.

as a stylistic variant of

> Some bricks are such that each of them is brick-r-touching each other one of them.

and sees that, in turn, as a stylistic variant of the very long sentence that results from unpacking both of the above definitions. The result involves only two predicates, '__ touches __' and '__ is a brick', and a whole lot of quantification and 'is one of' constructions. Since these predicates express properties, the Atomist can now apply the semantics.

Here are a few other examples of sentences that the Atomist must seek stylistic variants of, before applying the semantics. One type fairly explicitly involves notions of "having something in common".

> They are classmates.

is a variant of

> There is a class that each of them is in.

A good example from Thomas McKay:

> Companies that compete have common interests.

is a variant of

> For any companies, if each one of them competes with each other one of them, then for each one of them, and each other one of them, there is an interest that they both have.

Another type involves a covert common thing that the many are each connected to.

> They converse.

may be a stylistic variant of

> There is a conversation, of which each of them is an agent.

If so, the Atomist need not be committed to the truth of

> Some atoms conversed.

This example suggests that the Atomist might be happy to accept analyses of natural language on which verbs implicitly quantify over events. This analysis would take place at the level of "stylistic variation"—before the Atomist's semantics is called in. When plural noun phrases are the subjects for such verbs, the connection between the subjects and the events is typically distributive, as in the above. (It is not that they together were the agent of the conversation, but that they each were agents.) Thus we might reconsider

> John and Paul together lifted the piano.

as a stylistic variant of

> There was an event, it was a lifting, the piano was its patient, and each of John and Paul were agents of it.

If this is right, many typical natural language sentences involving plural subjects, even the "collective" uses, turn out, almost always, to involve a *distributive* use of the plural subject. The "distributive/collective" distinction remains, but becomes a distinction between two analyses, involving different distributions. (Is it that there is some event each of them is an agent of, or is it that they are each an agent of some event?)

5   Further reflections

5.1   What language does the Atomist speak?

We can imagine a language, call it *Atomese*, that sounds a lot like a natural language, and has a similar semantics to the one that Atomist proposes, but in which all singular terms refer to only one atom, and all plural terms refer only to single atoms (though they may refer more than once). So 'John' would be an unacceptable term, but 'Johnsatoms', taken to be a name that refers to (each of) John's atoms, would be acceptable, since every thing it refers to is an atom. Let us also constrain Atomese so that every primitive predicate in it expresses a property or relation. Any natural-language predicate that expresses a property can be a predicate of Atomese, e.g., 'is human', though we will have to allow it, grammatically, to form a sentence when connected with a plural term. Let us replace 'is' with 'be' so as to make this a little easier on the ear, so that

>   Johnsatoms be human

would be the Atomese way of saying what a natural language expresses with 'John is human'. Natural language predicates, like 'are children', that do not express properties, will not have correlates in Atomese. Let us also ensure that Atomese is a meta-language for our natural language. (Assume, for the sake of simplicity, that words and sentences of natural language are atoms, so that we can have singular terms for them. This makes our presentation easier, but is not necessary.)

Has the Atomist given us a way of *translating* natural language into Atomese? It depends what we mean by "translation". Here's what we can do:

Give the Atomist a suitable natural-language sentence. ("Suitable" means: from the fragment of natural language that we have given the resources to treat. No modal idioms, epistemic operators, self-reference, indexicals, etc.) There are two steps to the Atomistic analysis. First, we find a stylistic variant of it in which the only basic predicates will be ones that express properties and relations, and 'is one of'. Second, we apply the Atomist semantics to the result. The output of this process is a sentence of Atomese, which contains meta-linguistic terms, predicates like 'refers to' and 'assignment relation', and the Atomese correlates of

property-expressing predicates of natural language. (If the natural language sentence had quantifiers in it, the Atomese correlate includes quantifiers over relations.)

Thus

John is human

goes to

The atoms 'John' refers to be human.

and

Some things are each human, and collectively lift Mary.

goes to

For every assignment relation R, there is a 'they'-variant of it, S such that any atoms that bear S to 'they' be human, and such that the sum total of those atoms lift the atoms that 'Mary' refers to.

where

S is a 'they'-variant of R

abbreviates

S and R relate the same things to the same things, except perhaps for atoms that they relate 'they' to

and where

the sum total of those atoms

abbreviates

the atoms, atoms$_*$, such that each of atoms$_*$ is among some things that S relates 'they' to, and each atom that is among some things that S relates 'they to' is among atoms$_*$

If we write out the two-layered process in detail, we give a "theory of truth" for a fragment of the natural language. (Famously, we have to be careful about what we let into the fragment.) If we abstract away from the particular interpretations of names and predicates, we can arrive at an

Atomist "model-theory" for natural language. The only tricky part is to specify the notion of an arbitrary predicate-interpretation.[30]

This theory of truth, according to the Atomist, *gives correct truth-conditions* for the natural language sentences it talks about. So, if all you want out of translation is that, then we have a translation. Also, using the model-theory, the Atomist can propose a theory of *logical consequence* for (the fragment of) natural language: a sentence is a consequence of some sentences just in case every model of them is a model of it. So the Atomist "translation" process does not just give correct truth-conditions, it preserves (and maybe even explains) logical relations among sentences. If that's enough to count as "translation", then we have it.

But since the "translations" include meta-linguistic terms, we may be suspicious of the claim that they really *mean the same thing*.

Getting rid of the meta-linguistic terms

We may ask whether we can take a third interpretive step, finding a sentence of non-meta-linguistic Atomese that is equivalent with the sentence that comes out of the two-step process.

We can, if he have plural terms corresponding to the singular names of natural language and if any plural names of natural language are referentially equivalent with lists of singular names.[31] (A cute way to do this for natural-language singular names is to add 'atoms' as a postfix. A cuter way would be to just take the natural language name as it stands, but give it the new reference—make it semantically plural—and grammatical category. But this could lead to some confusion.) E.g.,

John is human.

goes to

---

[30]This can be done as follows. An interpretation-relation for the one-place predicates is a thing of the same kind as an assignment relation for plural variables: it relates each predicate many times, each time to some atom or atoms (collectively) from the domain of the model. Two-place predicates require a *three-place* interpretation relation: it relates each relation symbol many times, each time to some atoms (collectively) and some atoms (collectively), in that order. And so on up.

[31]The problem case is the plural name that refers infinitely many times. I do not see how to generate a non-meta-linguistic specification of a property that captures its referential profile.

> Johnsatoms be human.

where 'Johnsatoms' refers to (each of) the atoms that 'John' refers to. But

> Each of John and Paul is human.

does not go to the false

> Johnandpaulsatoms be human

where Johnandpaulsatoms refers to the sum total of atoms that 'John and Paul' refers to. We cannot systematically recover what is meant by natural language sentences involving 'is one of' with plural names like Johnandpaulsatoms. What we need involves a property:

> There is a property P, had by some atoms just in case they are referred to by 'John and Paul'. Any atoms that have P be human.

If we have a predicate in Atomese that expresses such a P, we can replace the free occurrence of the property-variable P, in the second sentence, with a predicate, and thus arrive at a non-meta-linguistic sentence that "translates" the original natural-language sentence.

We have such a predicate. Since we have 'Johnsatoms' and 'Paulsatoms', we have '(collectively) identical with either Johnsatoms or Paulsatoms', which does the trick.

If the natural-language sentence we started with involves plural quantifiers, we will end up with quantification over relations at the end of the third step:

> Some things are each human, and collectively lift Mary.

will end up at

> There is a property P such that any atoms that have P be human, and the sum total of atoms that have P collectively lift Marysatoms.

It makes sense we can always carry out the third step. (But see the one exception in footnote 31.) The fragment of natural language should have no greater internal expressive power than a meta-language in which we can express a "theory of truth" for it.

I do not know whether the Atomist should go so far as to say that what we get after the third step is a "translation" of the original. The Atomist can perhaps say that the proposition expressed by the Atomese sentence at step three is the same proposition as that expressed by the natural-language sentence we started with. But it also seems that there is some sense of 'synonymous' in which the original and final are not synonymous.

We may observe from this that the Atomist sees the expressive power of natural-language singular quantification (together with the natural language "part-whole" predicate) as tantamount to plural quantification in Atomese. And the power of natural-language plural quantification is tantamount to quantification over properties in Atomese.

'is one of' in Atomese

It should be noticed that Atomese contains and makes use of 'is one of'. This is surprising, since the natural language 'is one of' does not express a property.

But the Atomist does believe in a relation that is expressed by the Atomese predicate 'is one of': it is the relation that holds just when you put an atom x in its first blank spot and then put that atom, together with no or some other atoms, in the second blank spot. (There is even an expression of natural language that effectively expresses this relation: '__ is an atomic part of __'.)

In Atomese, unlike natural language, 'is one of' interacts with 'is/are identical with' in Atomese in a straightforward manner. In Atomese, for any grammatically plural terms t and s,

> t are (collectively) identical with (collectively) s

is, according to the Atomist, logically equivalent with

> every one of t is (identical with) one of s, and
> every one of s is (identical with) one of t

Thus we get the logical validity of the quantified biconditional:

> For any things (things$_1$) and any things (things$_2$): things$_1$ are (collectively) identical with (collectively) things$_2$ just in case every one of things$_1$ is one of things$_2$, and every one of things$_2$ is one of things $_1$.

Let us call this sentence "the Principle of Numerical Transparency", "PNT" for short. It fails in natural language. In natural language, the second of the two statements, corresponding to the right side of the embedded biconditional, is strictly logically stronger than the first: it entails the first, but not vice-versa. In natural language (at least in our fragment), the truth of the second statement guarantees the inter-substitutability *salva veritate* of t and s *in all contexts*, while the truth of the first only guarantees the substitutability *in contexts that express properties*. In Atomese, the statements are logically equivalent, and either one guarantees substitutability.

Now we might pose a question to the Atomist. The Atomist proposes a semantics for natural language such that when we abstract away from its particulars (holding fixed only the interpretations of 'is one of' and 'is identical with' and their plural correlates 'are among' and 'are identical with', and letting all other predicates and terms count as "non-logical") to generate a formally determined semantic notion of logical consequence, we find that PNT (as a sentence of natural language) fails. Yet the Atomist relies on PNT (as a sentence of Atomese) in developing the Atomist semantics for natural language. The question is: what entitles the Atomist to assume that PNT is valid in Atomese?[32]

The Atomist takes it that PNT is a fundamental, logical truth. The Atomist should not simply reply to our question that PNT expresses a straightforward logical insight into the notions of "being one of" and "identity". After all, the non-Atomist, speaking natural language, can presumably have just as much logical insight into these notions as the Atomist has, but would, on the basis of just this kind of insight, assert PNT as a sentence of natural language. This is basically the position of many philosophers who have thought about the logical aspects of 'is one of', 'is identical with', and their plural correlates.

But not exactly: the typical view (shared, it seems to me, by Yi, McKay, and Rayo) is that the condition

> things$_1$ are (collectively) identical with (collectively) things$_2$

is either nonsense, or is just *defined as* the condition that

> each of things$_1$ is one of things$_2$ and each of things$_2$ is one of things$_1$

---

[32]Gabriel Uzquiano drew my attention to something like this question.

On this view, there is no substantial logical principle here: PNT is at best a mere definition. (The notion of "definition" intended here is a deflationary one, so that a definition is effectively nothing more than a scheme of abbreviation—a stipulation about how some expression, otherwise absolutely meaningless, will be used. It is not the thicker notion of "definition", on which we can say that mathematicians struggled for years to find a good definition of continuity.) Our disagreement with this conception of PNT flows from the ideas of section 2.2. This disagreement is independent of Atomism: it is about whether the identity relation itself admits many things collectively into its argument places to form a proposition. If it does, and if we express such a proposition with the first condition, then the first condition is not *defined* (in a deflationary sense) as the second one. It may be that they are equivalent, and it may be that the second one *explicates* the first (or defines it in a thick sense, or bears some other such asymmetric relation to it).

I believe that PNT expresses an interesting (not merely definitional) principle, tied to the way 'is one of' connects with identity. Even if the ideas of section 2.2 are not accepted as a whole, I think this can be made plausible, by showing that it is plausible that there is a relation that behaves like I think identity behaves. Here is an attempt:

> There is a relation R such that for any things, (things$_1$) and any things, (things$_2$), things$_1$ (collectively) bear R to things$_2$ (collectively) just in case it is logically necessary that for any property, things$_1$ (collectively) have it just in case things$_2$ (collectively) have it.

We may then ask about how such a relation connects with 'is one of'. I think that if a philosopher who thinks of PNT as a mere abbreviation were to agree that there is such a relation, he or she would also think that the analog of PNT for it was logically valid. From here on, I will label this position *the standard line* about the logic of 'is one of' and identity.

Back to our main thread: the Atomist apparently cannot regard PNT as something that mere logical insight reveals to be true, since the non-Atomist also has logical insight, and hence would seem to be equally entitled to assert PNT—in natural language. Thus the Atomist must give some other explanation for why PNT (in Atomese) is valid.

Here are two ways the Atomist can respond. First, he could say that 'is one of' in Atomese and 'is one of' in natural language are very differ-

ent predicates, and that the apparatus of "plural quantification" in natural language is a different apparatus from the apparatus of "plural quantification". The Atomist could make this more clear by giving an abstract semantical theory for Atomese that shows the difference. This can be done. One way is to give an abstract semantics just like the one the Atomist gives for natural language, but in which, all singular terms refer to only one *thing*, and all plural terms refer only to single *things* (though they may refer more than once).[33] (Here we repeat what we said in introducing Atomese, but replacing 'atom' with 'thing'.) Another way is to give a semantics for Atomese along the lines of those given by Yi, McKay, or Rayo for natural language. The resulting formalized semantic logical consequence relation validates PNT. The standard line is right for Atomese, wrong for natural language. This response is somewhat unsatisfying, however, for there is something, intuitively, logically the same about the natural language 'is one of' and that of Atomese.

The other, and better response is to accept that the abstract semantics the Atomist has given to natural language is the one correct semantics for the logical notions of 'is one of' and 'identical with'. Then PNT is not valid after all, and we appear to have a fundamental logical disagreement with the standard line. *Yet PNT is valid if we restrict our quantifiers.* Atomese is not another language, with different kinds of quantifiers, and a different thing altogether meant by 'is one of', but, instead, a fragment of natural language, with exactly the same *kinds* of quantifiers and exactly the same 'is one of'. We would then agree with the standard line, but only when it is understood as about a language in which quantifiers are restricted in the right way.

To see what is going on here, observe first that the Atomist semantics for natural language validates a *restricted* version of PNT:

> For any things (things$_1$) *such that each of things$_1$ is an atom,* and any things (things$_2$) *such that each of things$_2$ is an atom*: things$_1$ are (collectively) identical with (collectively) things$_2$ just in case every one of things$_1$ is one of things$_2$, and every one of things$_2$ is one of things $_1$.

In general, if we take any sentence of Atomese, and replace it with a version in which the quantifiers are explicitly restricted to atoms, the result can be regarded as a logically equivalent natural language sentence;

---

[33]Extend the notion of reference to variables, relative to an assignment relation.

thus Atomese itself may be regarded as a fragment of natural language (in which the restriction of the quantifiers is orthographically repressed). Semantically, this restriction corresponds to using names and quantified variables whose reference and reference* are exactly the same.

If we take this second route, we may define the notions of "atom" and "atoms" in natural language, as follows:

> A thing, x, is an *atom* just in case: for any things, if they are (collectively) identical with it, then every one of them is identical with x.
> Some things are *atoms* just in case: every one of them is an atom.

Similarly, we can define "is part of" in natural language, as follows:

> Some thing x is a part of a thing y just in case x and y are (collectively) identical with y.

The essence of the metaphysical position of the Atomist then amounts to this:

> For every thing, there is at least one atom (thing such that any things identical with it are each it) that is part of it (such that the atom and it together are it).

Given *these* definitions, the logical position of the Atomist may not *contradict* the standard line, after all. The standard line sees PNT as a logical validity. The Atomist sees only a restricted version of PNT as a logical validity. But if the restriction is spelled out in terms of the above definitions, we see that those who maintain the standard line, from the Atomist's point of view, are simply assuming that the quantifiers of natural language are (implicitly, and universally) restricted to atoms (in the above sense). But the Atomist's metaphysics naturally leads him to think that our ordinary notion of "part" lines up with the one in the definition above, and is expressed in a language without such a restriction.

5.2   Ontological commitment

The Atomist would reject the idea that the restriction on the quantifiers of Atomese is a restriction of their ontological *range*. Relaxing the restriction does not increase their range, but only expands their *modes* of reference. And the whole point of the Atomist semantics for natural language, given

within Atomese, is that relaxing this restriction gives absolutely no new expressive power. But it does give the *appearance* of an inflation of ontology.

In Atomese,

> If there are two atoms, then there is some thing identical with (collectively) them, that is not identical with either one of them.

is not logically valid; in fact, its antecedent logically entails the negation of its consequent. In natural language, it is a validity. The Atomese equivalent of the natural language validity is

> If there are two atoms, then there are some atoms (collectively) identical with (collectively) them, that are not (collectively) identical with either one of them.

The basic structural relations on the "new things" that get "added" when the restriction is relaxed are given by Classical Atomistic Mereology. Thus, for example, the usual first-order renderings of sentences of the form

> There are exactly n things.

are invalid (logically false) unless $n = 2^i - 1$, for some positive integer i. Again, since natural language can be interpreted in Atomese, the apparent inflation is mere appearance: Mereology is ontologically innocent. Of course, you can't have something for nothing: the hidden source of the reductive power of the Atomist interpretation is in its use of properties, plurally quantified over, as outlined in section 3.2. The Atomist might be able to replace these properties with "extensional" entities—basically, sets that can bear the membership relation to many things collectively, as well as in the traditional manner. But that is a topic for another discussion.

References

[1] Donald L. M. Baxter. Identity in the loose and popular sense. *Mind*, 97:575–582, 1988.

[2] Donald L. M. Baxter. Many-one identity. *Philosophical Papers*, 17:193–216, 1988.

[3] Peter Van Inwagen. *Material Beings*. Cornell University Press, 1990.

[4] Henry Leonard and Nelson Goodman. The calculus of individuals and its uses. *Journal of Symbolic Logic*, 5:45–55.

[5] David Lewis. *Parts of Classes*. Blackwell, 1991.

[6] Adam Morton. Complex individuals and multigrade relations. *Noûs*, 9:309–318, 1975.

[7] Theodore Sider. Parthood. Forthcoming in a volume of *Topics in Contemporary Philosophy*.

[8] Alfred Tarski. Foundations of the geometry of solids. In John Cocoran, editor, *Logic, Semantics, Meta-Mathematics*. Hackett, 1983.

[9] Barry Taylor and A. P. Hazen. Flexibly structured predication. *Logique & Analyse*, 139–140:375–393, 1992.

[10] Gabriel Uzquiano-Cruz. Plurals and simples. *The Monist*, 87(3):429–451, July 2004.

[11] Byeong-Uk Yi. Is mereology ontologically innocent? *Philosophical Studies*, 93(2), February 1999.