There are at least three main approaches to thinking about the way parthood logically interacts with time. Roughly: the *eternalist perdurantist* approach, on which the primary parthood relation is eternal and two-placed, and objects that persist persist by having temporal parts; the *parameterist* approach, on which the primary parthood relation is eternal and logically three-placed (x is part of y at t) and objects may or may not have temporal parts; and the *tensed* approach, on which the primary parthood relation is two-placed but (in many cases) temporary, in the sense that it may be that x is part of y, though x was not part of y. (These characterizations are brief; too brief, in fact, as our discussion will eventually show.)

Orthogonally, there are different approaches to questions like Peter van Inwagen's "Special Composition Question" (SCQ): under what conditions do some objects compose something?[1] (Let us, for the moment, work with an undefined notion of "compose;" we will get more precise later.) One central divide is between those who answer the SCQ with "Under any conditions!" and those who disagree. In general, we can distinguish "plenitudinous" conceptions of composition, that accept this answer, from "sparse" conceptions, that do not. (van Inwagen uses the term "universalist" where we use "plenitudinous.") A question closely related to the SCQ is: under what conditions do some objects compose more than one thing? We may distinguish "flat" conceptions of composition, on which the answer is "Under no conditions!" from others.

One package of ideas popular among philosophers (e.g., embraced by Quine, Lewis, and Sider, to name just a few) combines the eternalist perdurantist approach to time and parthood with a flatly plenitudinous conception of composition. For this package, the formal theory of Classical Mereology (CM) is a perfect expression of its flatly plenitudinous "four-dimensionalist" metaphysics.

The project of this paper is to answer the question: what is the proper expression of the core idea of flat plenitude in the tenser's metaphysics of time? Or, a little more precisely, what are appropriate axioms to replace CM as the expression of flat plenitude in a non-eternalist setting, in which tense is taken seriously?

Though the focus will be on tense, formally similar issues arise for modality, so our results will bear directly on questions of modal mereology as well; throughout much of the following, we could replace the notions of a "time" and of something's being "always the case" with the notions of a "world" and of something's being "necessarily the case." Theorists interested in mereology, especially those who are at all attracted to flat plenitude, are likely to face analogs of the various questions addressed below, no matter what their approach to time. In the modal realm, we might ask, e.g.: Socrates might not have existed; would he then have been part of himself? Is there something that would have been, no matter how things might have gone, composed of Socrates and Plato? More than one? For each such thing, what could its mereological structure have been had Plato not existed? Do I have a mereologically coincident "world-slice," i.e., something of which I am part and which is part of me but could not have existed had things been different in any way? Even those who are not friendly to a tensed approach to time may be sympathetic with a non-reductive approach to metaphysical modality, and will thus face questions that are, formally, basically the same as those addressed by our tensed mereologies. And there may be other

---

[1]See [28].

important "modalities", e.g., metaphysical determinacy as used in a conception of vagueness in the world (as in [1]), with respect to which formally similar questions can be raised. It is thus hoped that those interested in systematic mereological metaphysics will find here something of use in approaching such questions.

The result of this paper's investigation is that there are a few different promising versions of tensed flat plenitude. They differ from one another in a number of ways but there are two main features that distinguish them from the expression of flat plenitude in Classical Mereology. First, on most of them it will turn out that most (but not all) mereological fusions change their parts. This may seem a surprising result, but it is not, once one has fully absorbed the fact that the formal theories behind the idea of a fusion make no mention of change, or its absence, or of the essential natures of the objects it speaks of. (van Inwagen's [29] is helpful in this connection.) Second, all of our candidates will suggest that fusions can temporarily mereologically coincide: it can happen that two non-identical things can have exactly the same parts, at least for a moment. Thus there can be failures of part-extensionality, understood as the principle that no two things can be made of the same parts. But other notions of extensionality are upheld: in all three candidates, it cannot happen that two non-identical things *always* have the same parts.

0.1   Preview, part one: a sketch of tensed flat plenitude

Let us now be a little more precise about our vocabulary. We will take "is part of" as given and primitive, except that we will speak as if it is analytic that everything is part of itself. We will take "overlaps" as defined in terms of "part:" x and y overlap just in case something is part of both x and y. We will need the notion of a "condition," for example, the condition of being a sitting cat. We well be more precise about what "conditions" are later; roughly, they will be open sentences.[2] For the moment, however, one might take a condition to be a "property" with the understanding that there are lots of properties, not just a few "natural" ones (e.g., there is the property of being a sitting cat, and there is the property of being either a sitting cat or identical with Socrates).

Now say that an object b is a "synchronic fusion" of a condition iff: both (1) every x that meets the condition is part of b; and (2) every part of b overlaps something that meets the condition. This is just the classical notion of fusion (except that we are taking ourselves to speak a tensed language). Say that an object b is a "diachronic fusion" of a condition iff: it is always the case that b synchronically fuses the condition.[3]

---

[2]Somewhat more precisely, we will want a "condition" to be an open sentence that may have more than one free variable, *together with a specification of a target variable*. For example, we will want to consider "conditions" like "y loves x", with "x" as target. This is because we want to consider, in effect, for each object that might be a value of the variable "y", the property of being a thing loved by that object. The point of this may be brought out by an example. We want as an instance of the plenitude principle, roughly this: that for every y, if y loves at least one thing, then there is a thing b such that b is a fusion of the condition (with respect to x) "y loves x" (i.e., a fusion of the condition of being loved by y).

[3]This notion is close to Sider's notion of "diachronic fusion" in [25]; it is very close to his notion of "minimal diachronic fusion." But of course the notion here uses tense and the notion of a condition (as opposed to a tenseless language and the set-theoretic constructions that are

For example, consider the condition of being a sitting cat. Suppose that now, b synchronically fuses that condition, and c diachronically fuses it. Then: we know that everything that is (now) a sitting cat is (now) part of both b and c. We know that (assuming parthood is transitive) every part of b now overlaps c and vice-versa. We also know that if some cat began sitting yesterday, and continued to sit continuously until now, then it was part of c that whole time. We do not know whether it was part of b until now; from what we have assumed about b, it may or may not have been. Similarly, if Tibbles is now a sitting cat, but will stand up in a few minutes, we know that (given that never does a standing cat overlap a sitting cat), Tibbles will then cease to be a part of c. We can't infer this about b.

Using our vocabulary, a variation of van Inwagen's SCQ is then: which conditions have a synchronic fusion? This question is extremely close to van Inwagen's original version of the question.[4] But a different variation is: which conditions have a diachronic fusion? This question is significantly different from van Inwagen's original question. And one can ask two questions about flatness. Which conditions have more than one synchronic fusion? Which conditions have more than one diachronic fusion?

Using our vocabulary, here is a preview of the candidates for tensed mereology presented in this paper. The first of the three candidate axiom sets can be summed up rather briefly, as the axioms *parthood is transitive*, *parts and wholes exist*, and *any condition has exactly one diachronic fusion.* The last proposition is clearly an expression of flat plenitude, though it is also clearly quite different from flat plenitude as expressed in CM. The second candidate axiom set is geared toward the possibility of a robust distinction, drawn within the tensed setting, between *endurants* and *perdurants*, where to be an endurant is simply to never have non-existent parts and to be a perdurant is to be a non-endurant. This axiom set can be supplemented to generate the result that nothing endures, or that everything endures (in which case it collapses into our first candidate), or that there is a mixture of endurants and perdurants. The third candidate is a formally smooth generalization of the other two, but it is less clear how well motivated, metaphysically, it is.

The primary goal of this paper is to explore the technical details of some reasonable options, rather than to argue, on metaphysical grounds, for any particular one. Nonetheless, it is appropriate to say at least a little about what might be attractive about the options explored, and we will do so along the way. We will also discuss some alternatives, some of which are suggested by views found in the literature, and reasons against them.

0.2 Preview, part two: a version of parameterist flat plenitude

It is worth giving a sketch of how one might axiomatize flat plenitude within the *parameterist* conception of time and part. On this conception, we may express flat plenitude within a first-order language that purports to quantify over all

---

Sider's "assignments").

[4]Formally, one can show them to be equivalent under the assumption of a few plausible principles concerning plural logic and the parthood relation.

that there ever was, is, or will be, as well as times themselves (however those are conceived), and whose primary mereological predicate is the grammatically three-placed "x is part of y at t."[5]

Parameterists include both *relationists*, who hold that parthood is a relation that holds between or among an object, an object, and a time, and *adverbialists* who hold that while the parthood relation is itself two-placed, it nonetheless holds in a manner that is expressed by the "at t" component of the predicate. The difference between relationists and adverbialists is relevant to discussions of what would appear to be intrinsic properties, such as, let us say, the properties of being bent and being straight. The relationist about these properties holds that being bent turns out to be a relation that an object may bear to a time, while the adverbialist holds that being bent really is a property, but that an object *has* it (if it does) in a temporally qualified way.[6] For our purposes, the difference between relationism and adverbialism does not matter; we are lumping together as *parameterists* all those whose fundamental parthood predicate is grammatically three-placed, one of the places reserved for a time, independent of the conception of the "metaphysical interaction" among the "components" indicated in the three places.

For ease of presentation, let us assume that there are primitive notions of something's being a time, and something's existing at a time, and let us assume a certain minimal amount of set theory to be accepted and expressible in our language. All these assumptions facilitate axiomatization, so that the set-embracing parameterist framework makes it relatively simple to express flat plenitude.

For the purposes of this paragraph only, say that something is an "object" if it is not a time or a set (or take "object" as another primitive). Then a candidate axiom set is given by (the universal closures of) the following three axioms:

> Axiom 1: If x is part of y at t, then: t is a time, and x and y exist at t, and x and y are objects.
>
> Axiom 2: If x is part of y at t and y is part of z at t, then x is part of z at t.
>
> Definitions: If f is a function from the set of times to the power set of the set of objects, then say that f is "suitable" iff: (1) for every time t, $f(t)$ is a set every member of which exists at t; and (2) for some time t, $f(t)$ is non-empty. If e is a set of objects, say that an object b is a "synchronic p-fusion" of e at t iff: (1) every member of e is part of b at t; and (2) every part of b at t has a part at t in common with some member of e. If f is a suitable function, say that object b is a "diachronic p-fusion" of f iff, for every time t, b is a synchronic p-fusion of $f(t)$ at t.
>
> Axiom 3: Every suitable function f has exactly one diachronic p-fusion.

---

[5] It is worth noting that, as discussed below, the tenser, with certain assumptions, may also speak of "times" and may even define a grammatically similar three-placed predicate. But the tenser's three-placed predicate is derivative, not primary, and things said with it are to be regarded as analyzable into things said without it.

[6] Johnston [15] is a seminal expression of adverbialism, Lewis [19] is an important critical discussion, and Wasserman [31] provides a useful overview of the motivations and difficulties for both relationism and adverbialism.

This expression of flat plenitude is basically equivalent to one (out of a few) entertained by John Hawthorne [12]. A different expression of flat plenitude within a parameterist treatment of time is advocated by Judith Jarvis Thomson in [27]. Something approaching Thomson's idea can be had by revising the definition above of a "suitable" function by adding a third clause: for any object x, if for some time t, $x \in f(t)$, then for every time t at which x exists, $x \in f(t)$. Call this axiom set the "weakened set." (Unlike the original set, the weakened set does not seem to yield some of the structural principles Thomson desires, for example, that each object is part of itself when and only when it exists. So recovering her conception would require a small amount of supplementation.) It is worth noting (emphasizing a theme of Hawthorne's) that something equivalent with the original axiom set can be arrived at by adding, to the weakened set, an axiom stating that for each object that exists at a time, it has an "instantaneous temporal part" at that time.[7]

This expression of flat plenitude, within the parameterist conception, corresponds tightly with the first candidate tensed mereology we will consider, as axiomatized by the set TM1′ below. But our candidate will be stated in a tensed language, in which the only primitive predicate (besides identity) is a two-placed relation-symbol, and will make no use of set theory nor of a primitive predicate for being a time. And our second and third candidates will diverge more significantly.

1   Preliminaries

1.1   Flat plenitude in Classical Mereology

Philosophers have sometimes taken "mereological fusions" to be things that *by definition* have certain properties that, in fact, the bare theory CM is neutral on. I believe that many have thought that, *by definition* a fusion has one or more of these three features: (1) it does not gain or lose parts; (2) it could not have had any parts other than those it in fact has; and even (3) it has its "identity conditions" determined by some ultra-thin kind or sortal like *mere aggregate of stuff/things*. Relatedly, Hugh Mellor writes "I will not identify any thing or event with the so-called *mereological sum* of the things or events which are its parts; since this entity, if it exists at all (which I doubt), exists if its parts do by mere definition." p.87 [21]. Though Mellor's notion that sums are supposed to *exist* in part "by definition" may not strictly require any of the above three features, it is suggestive, I believe, of all three. But the bare theory CM says no such thing; again, van Inwagen [29] is clarifying in this regard.

While it may seem close to nitpicking to point out that the bare theory says none of these things about fusions, I hope to make a more substantial point here, to the effect that when the idea of flat plenitude is expressed in a tensed approach

---

[7]Say that y is "instantaneous temporal part of" x at t if y exists at t and only at t, and x is part of y at t and y is part of x at t. The equivalence is straightforward to show, once one has shown that each axiom set entails that every object is part of itself when and only when it exists. (See the remarks below about TM1′.) If g is an arbitrary suitable function in the original sense, consider the function f such that for each time t, $f(t) = \{x : x \text{ is an instantaneous temporal part at } t \text{ of some } y \in g(t)\}$.

to time, it is natural to fix on a different, but closely related, conception (or conceptions) of "fusion" from that used in CM, e.g., the notion of diachronic fusion given above. Given a plenitude of diachronic fusions, it is common for a fusion to lack property (1): many fusions gain or lose parts, though many fusions don't. Moreover, if the basic drift of this paper is right, it is plausible that when we place the idea of flat plenitude in a context in which we consider metaphysical possibility, it should express itself with yet another notion of "fusion," where, in the resulting theory, similarly, many but not all fusions will lack property (2): say that b is a modal fusion of a condition iff b necessarily is a diachronic fusion of that condition. (I will not here address whether property (3) is clearly affected by these sorts of re-expressions of flat plenitude.)

Indulging in a historical fantasy about CM, I imagine its original formulators (the likes of Lesniewski, Tarski, and Leonard and Goodman) to have combined what I am thinking of as the "core," with a metaphysical outlook that is eternalist in character, on which time is a fourth dimension (as is suggested by special relativity) and on which metaphysical possibility is an antiquated essentialism to be ignored in science. Thus CM is cast with a two-place part-whole predicate in an untensed, nonmodal language. If we find this metaphysical outlook in some way lacking, we should consider the result of letting flat plenitude express itself in a metaphysical setting we find more congenial. Though we might disagree with the resulting theory, I believe that the resulting theory will be more interesting to consider than bare CM.

Let us now be a bit more precise about what the conception of parthood embodied by CM is. To abbreviate, let us use the symbol "$\leq$" for the part-whole relation.

Informally, one can divide the conception into two aspects: one *structural*, and the other *existential*. Key structural ideas include, but are not limited to, these: $\leq$ is reflexive, transitive, and anti-symmetric. The key existential idea is roughly that *for any condition, it has a fusion.* Intuitively, the fusion for a condition should somehow gather up, but minimize. That is, it should obey these two roughly stated constraints: (1) everything that satisfies the condition is part of the fusion; and (2) the fusion is "the smallest" thing that meets constraint (1).

Alternatively, one can divide the conception into one structural idea, that parthood is transitive, and one compositional idea, that composition is flatly plenitudinous: every non-empty condition has one and only one fusion. The latter idea is, plausibly, both structural and existential.

These two divisions of the conception are not really in competition; they are two ways of looking at the same thing. The two informal presentations correspond to two different axiomatizations of the same formal theory.

1.2   Formalism

Let us now review how these informal ideas may be formalized. We will consider two equivalent axiomatizations of what we may call "pure formal CM:" CM cast in a language that talks about nothing other than parts and identity.

We use a first-order language $\mathcal{L}(\leq)$ with identity and the two-place predicate $\leq$

and no other predicates. Define "x overlaps y" ("$x \circ y$" in symbols) as "something bears $\leq$ to both x and y" ("$\exists z(z \leq x \land z \leq y)$"). The axioms of CM1 are these

Structural constraints: The $\leq$ relation is reflexive, transitive, and anti-symmetric. Further, it has what we will call the "strong supplementation" feature:[8]

(SSUP) $\forall x \forall y(\forall z(z \circ x \rightarrow z \circ y) \rightarrow x \leq y)$.

Existential constraint: For any non-empty condition, there is something that fuses that condition. To be an object b that fuses a condition is (as with our notion of synchronic fusion) stipulated to mean: to be such that (1) every thing that meets that condition is part of b; and (2) every part of b overlaps something that meets the condition. The first clause corresponds to the intuitive idea of "gathering up," and the second to the intuitive idea of "minimizing." Formally, for any well-formed formula (wff) ϕ_x in which the variable x may be free, find two variables (here b and y) that have no free occurrences in ϕ_x, and let

$Fu(b, [x|\phi_x])$

abbreviate

$\forall x(\phi_x \rightarrow x \leq b) \land \forall y(y \leq b \rightarrow \exists x(\phi_x \land y \circ x))$.

We then have an axiom scheme: for any ϕ_x (and b and y as above) the universal closure of

$\exists x \phi_x \ \rightarrow \ \exists b \ Fu(b, [x|\phi_x])$

is an axiom.

Again, let us call this axiom set CM1.[9] "Pure formal CM" is the set of its theorems. (In general, we also want to consider other theories that arise from expanding the stock of primitive expressions in the language, and adding every instance of the fusion scheme that is statable in the expanded language. But, to keep things simple, in our formalisms, we will focus on "pure" theories in which the vocabulary is kept to a minimum.)

There are many other equally natural ways to axiomatize pure formal CM, some that use other but equally natural definitions of fusion. The other main axiom set of interest for us, which we call CM1′, is Tarski's surprisingly short and equivalent axiomatization (expressed in set-theoretic form in [26]). It says that $\leq$ is transitive, and that every non-empty condition has a unique fusion, as in:

$\exists x \phi_x \ \rightarrow \ \exists! b \ Fu(b, [x|\phi_x])$.

Above, it was suggested that the conception of parthood given by CM is a package of structural ideas about parthood (reflexivity, transitivity, etc.), and one existential idea (plenitude, the idea that for any non-empty condition, something fuses that condition). The flatness in flat plenitude then emerges as a result of the structural ideas. The first axiom set, CM1, fits this conception nicely. In CM1′, by contrast, there might seem to be fewer structural ideas (transitivity alone). Yet the equivalence of the two axiom sets reveals that the uniqueness

---

[8]The name comes from the common name of a closely related axiom-or-theorem found in the literature.

[9]Reflexivity can be dropped, since it can be deduced from (SSUP) alone.

clause—the flatness—built into the CM1′ fusion axiom scheme, encodes, so to speak, the additional structural features highlighted in CM1.[10] CM1 neatly separates the structural from the existential. But the idea of flat plenitude has its own naturalness, and so it is hard to select either formal axiom set as more clearly or explicitly corresponding to the informal ideas behind it.[11]

1.3   Schemes vs. auxiliaries

van Inwagen posed the Special Composition Question using plural vocabulary. Instead of asking when a condition (or property) has a fusion, he asked: given some things, when do *they* compose something? And his definitions of "fuse" and "compose" make essential use of the plural. There is a reason why we have not been using plural quantifiers and variables, but have instead been speaking of the fusion of a *condition*.

First, let us consider some technicalities. In CM1 (above) the idea of plenitude is expressed by way of an axiom *scheme*. But in some formulations of CM, the scheme is replaced with a single axiom. This is made possible only by interweaving CM with another theory, itself with a kind of "comprehension" scheme, typically either set theory or the theory of plural quantification. For example, David Lewis's expression of CM in [18] uses plural quantification and has a single fusion axiom (let us call it (PFP) for plural flat plenitude) that "does all the work" of the Tarski-style schematic axiom above:

for any xs, $\exists! b(\forall x(x \text{ is one of } x\text{s} \to x \leq b) \,\land$
$$\forall y(y \leq b \to \exists x(x \text{ is one of } x\text{s} \land y \circ x))).$$

Of course, nothing comes for free. The (PFP) axiom manages to do all this work only because it gets to help itself to the infinite resources of an axiom scheme of comprehension in the logic for plural quantification. (And the scheme must be taken to have instances involving the $\leq$ symbol.) The scheme is as follows: for any wff ϕ_x in which the plural variable "xs" is not free, the universal closure of the following is an axiom

$$\exists x \phi_x \to \exists x\text{s}\, \forall x(x \text{ is one of } x\text{s} \leftrightarrow \phi_x).$$

Technically, we could consider a Lewis-style axiomatic theory CMP (Classical Mereology with plurals), cast in a language $\mathcal{L}^P(\leq)$ with identity and "is one of" as logical relations, and with $\leq$ as the only non-logical primitive. The axioms of

---

[10]The success of this encoding turns on the definition of fusion we used. This definition can be motivated by the intuitive idea that a fusion should "gather up" the things that meet a condition, but "minimize," as discussed above. Other natural definitions of fusion are in a sense weaker: to yield the the other structural principles of CM1, one requires more than transitivity and the flat plenitude of these other sorts of fusions. See [13] for details.

[11]Other equivalent axiom sets do not so clearly factor out into structural and existential components. For example, a somewhat curious result is that one can replace the seemingly structural (SSUP) axiom from CM1 with the following, seemingly existential, Strong Complementation axiom, and yet reach exactly the same theory:

(SC) $\forall x(\neg \forall y\, y \leq x \to$
$$\exists z(z \wr x \land \forall y((y \wr x \to y \leq z) \land (y \wr z \to y \leq x))))$$

where $x \wr y$ means that x and y are disjoint, that is, $\neg\, x \circ y$. (Reflexivity may be deduced from (SC) alone.) A point in favor of (SSUP) as getting at the essence of the structural requirement is that (SSUP) is weaker relative to transitivity alone: (SC) and transitivity together imply (SSUP), but transitivity and (SSUP) do not together imply (SC). An analogue of (SC) will come into play for our third candidate tensed mereology.

CMP then would be two sentences: the transitivity of $\leq$ and the (PFP) axiom. The theory determined by these axioms would then be the consequences of their conjunction in a suitable plural logic, crucially including the comprehension scheme.[12] Given such a logic, one can easily see how any instance of the Tarski-style fusion scheme, from CM1′, can be derived from the corresponding instance of plural comprehension, together with (PFP). Thus, while CMP is not a theory in the language $\mathcal{L}(\leq)$, it is in an expansion of that language, and every axiom of CM1′ is implied by CMP with the plural logic. It appears also that the CMP theorems that are in $\mathcal{L}(\leq)$ should be the same as the theorems of pure formal CM (the theorems of CM1), so that CMP and CM1′ are in this way equivalent.

Less technically, it is natural to associate with the Lewis-style axiomatization the following definition of "b fuses xs:"

$$\forall x(x \text{ is one of } xs \rightarrow x \leq b) \wedge \forall y(y \leq b \rightarrow \exists x(x \text{ is one of } xs \wedge y \circ x)).$$

Then the Lewis-style fusion axiom can be expressed in the delightfully short sentence "Any things have a unique fusion." And this vocabulary is very close to that used by van Inwagen in discussing the SCQ and related issues.

As nicely simplifying as the plural vocabulary may be, I want to urge again that the idea of flat plenitude can express itself differently in different metaphysical settings. In particular, the idea of plenitude, that *any condition* has some kind of fusion (something that gathers up and minimizes), can manifest itself in different ways depending on how the highly abstract notion of "any condition" is made out in the different settings. The plural quantification axiom of Lewis is suitable to replace the scheme only because in the perdurantist setting favored by Lewis, conditions can be "simulated" by pluralities. In this setting, if two conditions happen to determine the same things (that is, every one of the things that satisfy the first is one of the things that satisfy the second, and vice-versa), it is desired that they have the same fusion. But this kind of "extensionality" is an undesirable result in the tensed setting, given the "intensional" aspect that a condition has in the tensed setting: what a condition determines (now) (at such and such time) does not settle what it determines at other times. My suggestion, to put it roughly, is that "fusing" should track intension, not mere extension; but pluralities (as usually conceived) are rigid (once one of xs, always one of xs), and so it will not be possible in general to replace the instances of a fusion axiom scheme with a single axiom governing pluralities. In general, it is only on a fully "extensionalist" metaphysics, so to speak, that "any condition" can be replaced with "any things."

Moreover, though this must remain informal and somewhat vague, I would like to suggest that the use of schemes is more conceptually fundamental than the use of auxilliary devices like plural quantification, even with metaphysical theses like Lewis'. For one thing, schemes are "sneaked in" via their use in the auxilliary devices; e.g., in the plural comprehension scheme in the Lewis-style system above. What this suggests is that the expression of flat plenitude as an axiom scheme is what sets the target: the auxiliaries are up to the job only if they suffice to generate what the scheme generates.

We must grant, of course, the technical point that the instances of a scheme in any fixed countable language are countably many, and hence plausibly do not

---

[12]See Yi [32] and [33] for a careful exposition of such a logic.

themselves capture all that we intend with our "for any things," let alone with "for any condition" or "for any property" (with an informal use of "condition," or "property"). (Proof: If there are even *countably* infinitely many things, then there are *uncountably* many thingses, so to speak, and so the (countably many) instances of the scheme do not cover them all.) But it is plausible that, in our case, one should understand commitment to the scheme to be commitment to its instances not just in one's current language, but in any expansion of that language one might come to speak. (This point is inspired in part by McGee's argument for, and conception of, the epistemological and logical centrality of the (first-order) axiom scheme for mathematical induction, as opposed to any single axiom or any set of instances of the scheme, in [20].)

2 Tense and flat plenitude

Let us use "the tensed treatment of time," or "(T)" for short, as an imprecise term for a package of methodological, semantic, and metaphysical views. A key, fairly concrete, manifestation of the treatment is the methodological view that *The best language for expressing the facts is tensed.* A key semantic view is that the truth-values of *fully formed* sentences (or genuine "propositions") may change: "Socrates is sitting" does not contain any hidden parameters like "at noon." It is *fully* formed and yet may change its truth value. And a key metaphysical view is that past, present, and future reality are not metaphysically parallel pieces of reality that are included in a larger piece of the same basic kind; rather, the past is how the one reality *was*, the present is how it *is*, and the future is how it *will be.* The views interlock: for example, we may now say truly that though Socrates is not sitting, Socrates was sitting. The current truth of "Socrates was sitting" is aligned with the past truth of "Socrates is sitting" and expresses that reality was a way that it now is not. And it is not that Socrates is sitting in one part of reality, but is not sitting in another part; rather, reality *includes* no sitting Socrates, though it *included* a sitting Socrates. (The package is close to what Sider calls *presentism* in Chapter Two of [25]. *Tense realism* might be a slightly more exact term for just what I'm after, as I take what I'm after to be consistent with the view Fine calls *fragmentalism,* which is supposed to differ from some main forms of presentism in holding, against them, that no time is privileged as the one true "now." See [11] for fragmentalism and the idea that realism about tense could take different forms.)

2.1 First candidate: informal presentation

The project is to consider how flat plenitude might express itself in (T). We make the somewhat vague assumption that we are to be guided by CM as much as possible: we are looking for a *CM-like* expression of flat plenitude. For example, we will want something identical to or analogous to the transitivity of parthood. But we cannot keep everything, of course, and, in particular, we will end up considerably modifying anti-symmetry. We will eventually consider three main candidates for a CM-like expression of flat plenitude in (T). In this section, we prepare for an explicit and formal discussion of our candidates by considering, informally, some of the issues they respond to, highlighting the responses that

lead to the first candidate.

Now one key metaphysical component of (T) is that existence may change: what existed may not now exist, and what exists may not have existed.[13] (Further, but closely related, is the idea that the "range" of the most inclusive quantifiers changes: it may be that absolutely every thing is such that it was never a dinosaur, even though there was something that was a dinosaur.) Accordingly, the expression of the core in (T) needs to have something to say, structurally, about whether there are existential constraints on the $\leq$ relation. The most obvious suggestion (embraced in parameterist form by Thomson and Hawthorne in [27] and [12]) is that the $\leq$ relation can hold only among existing things. For now, let us take this suggestion. (Below, the first of our three formalized candidates will adopt this policy, while the second will adopt a weaker echo of it, and the third will basically drop it.) Another question is what the existence conditions for a fusion are. The most natural idea is that a fusion for a condition exists when and only when there exists something that meets that condition.

Next, let us begin to consider how CM's central structural requirements on $\leq$, such as transitivity, might express themselves in (T). The most natural suggestion seems to be that we take them simply to hold always: $\leq$ is always transitive, always reflexive, etc. (But we must reconsider this momentarily.) To say that $\leq$ is always reflexive is to say that it always holds that: everything bears $\leq$ to itself. It is not to say that everything always bears $\leq$ to itself. The latter is basically incompatible with the requirement that the $\leq$ relation holds only among things that exist. (There was a dinosaur that, now, does not exist and now does not bear $\leq$ to itself.) Below, we will use expressions like "[always, transitivity]" to mean "it is always the case that for any x, y, and z, if $x \leq y$ and $y \leq z$, then $x \leq z$." Let us accept [always, reflexivity] and [always, transitivity] for now; we will return to the others later.

Let us next consider how plenitude might be made out. On the tenseless conception, a fusion of a condition is something that both has all the things that meet the condition as parts, and is such that every part of it overlaps something that meets the condition. Above, we used the same words, taken to be tensed in our definition of what it is to *synchronically* fuse a condition. But we will also defined the notion of *diachronic* fusion: a diachronic fusion of a condition is something that *always* synchronically fuses it. A natural suggestion for the expression of plenitude in (T) is that every suitable condition has a diachronic fusion (where a condition is suitable iff it is not always empty; i.e., it is suitable iff at some time, at least one thing satisfies it). This is the suggestion explored in full formal detail in our first candidate axiom set. Recall the basic point, that if we know that c now diachronically fuses the condition of being a sitting cat, then we know that it is always the case that every sitting cat is part of c. But knowing only that b now synchronically fuses the condition of being a sitting cat tells us nothing about what things were and were not part of b, nor what things will and will not be part of b.

---

[13]This is not to say that there could not be a (T)-like view on which existence cannot, does not, or at least might not, change. But such views are easy enough for us to accommodate if we know how to deal with views on which existence does change.

2.1.1 Rejected alternatives for plenitude

Before moving on, we should discuss a different and obvious candidate for the expression of the plenitude principle in (T). We might make it go similarly with what we have done with reflexivity. The natural suggestion was that reflexivity is expressed as the principle [always, reflexivity], i.e., it is always the case that everything is part of itself. To apply the parallel to plenitude, we would get, roughly: *it is always the case that*, for any (suitable, i.e., non-empty) condition, there is a fusion for it, a thing whose spread over reality matches the (current) manner in which the condition carves up reality.[14] It is natural then to conjoin this with structural axioms that are basically just the results of saying "always" before each of the classical structural axioms, and an [always, flatness] axiom, so that no condition ever has more than one fusion. But the resulting theory, I will now argue, is far *too weak* to be a plausible CM-like expression of flat plenitude.

Flat plenitude would have it that at each time there is one and only one fusion for each non-empty condition. Suppose that x now fuses the students, and that Jill is now a student. Suppose further that she will be a non-student; will she then be part of x? Either answer is compatible with the axioms under consideration. We know that there will then be something y that fuses the things that are then students, and we may assume that there will be something z that then fuses the things that are now students. Jill will be part of z and not part of y, but that $x = y$ is compatible with our axioms, while so is $x = z$ (though of course their conjunction isn't). Indeed, it is compatible with our axioms that x only exist for an instant; but it is also compatible that x always exists, and at every time other than now, has everything as part of it.

I suggest that the mereology itself should settle such questions about the cross-temporal behavior of fusions; in this sense, the axioms under consideration are too weak. One way to augment them so that they settle such questions would be to adopt a structural requirement to the effect that $\leq$ relations never change (at least when both relata exist), comporting with Roderick Chisholm's doctrine of *mereological essentialism*, as defended in [6] and [7]. Then the cross-temporal behavior of the fusion would be greatly constrained by its current behavior. This seems to be the idea had by those who take "mereological fusions" to be things that, by definition, never change their parts.

But this requirement is ultimately rather unmotivated, if the $\leq$ relation is meant to be something like either an *analysis* or an *explication* (in the sense of Carnap [5] p. 7 ff. or Quine [23] p. 258 ff.) of our ordinary talk of parts and wholes. Given that the expression in (T) of this talk has it that things routinely gain

---

[14]Actually, there are some significant technical wrinkles here. When we axiomatize, we have a number of choices for exactly how to construe the instances of the fusion axiom. First, there is the question of what language the fusible conditions are in: CM's axioms do not have tense operators in them anywhere, but we might want to have, for example, an axiom that says that if there are any things that have always existed, then there is a fusion for the condition "x has always existed." Second, there is the question of how we "close" axioms involving conditions that include free variables other than the target (which we have been always representing as x). The very simplest approach would be to just plop the "it is always the case" operator in front of the universal closure of a classical axiom. But we might, instead, plop the "always" operator on and then universally close. Or we might "modally" close, interweaving "always" with the universal quantifier. These choices are not generally equivalent with one another.

and lose parts, the $\leq$ relation should be similarly flexible, unless allowing this flexibility somehow makes it impossible to give a satisfying explication.

Relatedly, and to interweave an earlier theme, note that, in general, the notion of a flat plenitude of *diachronic* fusions is not easily converted into a proposition that could be expressed in a single sentence, in the way that Lewis' plural quantification allows his fusion axiom to be a single sentence.[15] We cannot simply say that (it is always the case that) any things have exactly one diachronic fusion. For, on my candidate, to get the diachronic fusion for a condition, we do not simply take the objects that meet the condition at one time, and then fuse *them*. Nor do we look across all time, and gather up the things that *ever* meet the condition, and simply fuse all of *them*. Rather, the fusion for a condition gains and loses parts as different things come to meet and cease to meet the condition. We follow the condition, not the identities of some objects. (Of course, we allow conditions that are themselves rigid and determined simply by the identities of objects, so that *among* our fusions are some that effectively take some objects and fuse *them* across all time.)

Now, we are considering the suggestion that flat plenitude in (T) be expressed as basically the principle that it is always true that each non-empty condition has exactly one (synchronic) fusion. We have seen that this idea raises but does not settle questions about the cross-temporal behavior of fusions, and hence needs supplementation. And we have seen that it is not promising to attempt to settle these questions by adding an axiom that nothing ever changes its parts. An initially more promising approach to finding something to add is suggested by a view in the neighborhood of views expressed in the metaphysics literature by Michael Burke (e.g., [3] and [4]) and Michael Rea (e.g., [24]).[16] The basic idea is that a robust theory of sortals or kinds is exactly what is needed to settle such matters: briefly put, we may "trace" a thing across time by looking at the key sortal or kind that it falls under. For example, when statue and lump *appear* to mereologically coincide by being made of the same clay, they cannot actually coincide, on pain of being identical. On this view, the kind *statue* takes over. When the statue comes into existence (for the first time), it is the fusion, let us suppose, of the condition "being one of these molecules." But since it did not exist before, there was never anything that earlier fused that condition and now does. But (assuming none of the molecules just came into existence) just earlier, something fused that condition; so, that thing, plausibly a lump of clay, now has either ceased to exist or has lost or gained at least one molecule.

Even if one is willing to accept this mildly unintuitive result, this sort of example is merely the tip of the iceberg. Given the astonishing miscellany that the sort of plenitude we are looking at provides, at a time, it is hard to believe, for metaphysical reasons, that cross-temporal existence and identity questions have the sorts of answers the current view requires them to have. Consider the fusion

---

[15]I assume that pluralities cannot change their "members." If we worked against a backdrop of a property theory, on which it is a theorem, for example, that there is a property s such that it always holds that for any object x, x is a student iff x has s, then we might be able to deduce our fusion axiom scheme from a single sentence saying something like "every not-always-empty property has a diachronic fusion." Another possible source of auxilliary entities would be a set theory on which sets can change their members.

[16]It should be emphasized that the view entertained and rejected below is not taken to be that of Burke or Rea; rather, it is what results from adding an idea of theirs (the "dominant sortal" or "kind" idea) to the current take on tensed flat plenitude.

of a cat and the tree it is perched in. What kind of thing is that? Will that fusion continue to exist for some time? If so, will it remain a fusion of that cat and that tree? Will it cease to exist when the cat comes down out of the tree? Does it only exist for an instant? Parallel questions arise for such fusions as: the fusion of all surfaces of things in your visual field; the fusion of all the cars and drivers stuck in one traffic jam; the fusion of a mother and her unborn child; etc., etc. What is hard to believe, I suggest, is that the space of sortals or kinds is robust enough to provide answers; but on this view, the answers are *out there*.

Thus I conclude that the idea that flat plenitude in (T) should express itself as "it is always the case that each non-empty condition has exactly one synchronic fusion" is likely to leave certain questions about the cross-temporal behavior of fusions unanswered, or poorly answered. I have suggested that we consider instead something more like the following form of plenitude: it is always the case that every suitable condition has a diachronic fusion. This suggestion, when coupled with a few other simple principles, has the enormous advantage that it writes in answers to these questions. Tell us which condition is being fused, and, to the extent that we know which things satisfy the condition at each time, we know "where" that fusion is at that time. If we consider a fusion of the condition "x is identical with the cat or x is identical with the tree," we know that it exists when and only when either the cat or tree exists, and we know that you overlap it at a time just in case you then overlap either the cat or the tree. If we consider a fusion of the condition "(x is identical with the cat or x is identical with the tree) and both cat and tree exist," we get another set of clear answers; and yet another set of clear answers for a fusion of the condition "(x is identical with the cat or x is identical with the tree) and the cat is in the tree."

2.1.2   Anti-symmetry and extensionality

Now suppose we adopt what I have said so far regarding a CM-like expression of flat plenitude in (T): non-existent things are never parts or wholes, reflexivity becomes [always, reflexivity], transitivity becomes [always, transitivity], and plenitude is expressed through the existence of diachronic fusions. We next consider what we should do with the other structural requirements on parthood in CM1, namely anti-symmetry and strong supplementation. An obvious thing to do is to consider the axioms [always, anti-symmetry] and [always, strong supplementation]. But this runs us into a serious difficulty. For consider a (diachronic) fusion of the condition on x that "(x is Socrates and Socrates is sitting) or (x is Plato and Socrates is not sitting)." Suppose β is such a fusion. Then, when Socrates and Plato are sitting at dinner, β exists and it should hold (then) that everything that overlaps Socrates overlaps β and vice-versa. By strong supplementation, Socrates and β then bear $\leq$ to one another. By anti-symmetry, they are then identical. But later, when Socrates stands, β will then (by similar reasoning) be identical with Plato, yet Socrates won't be identical with Plato, so Socrates and β are then non-identical. I take this result to be unacceptable: once identical, always identical, certainly if "both" exist.[17]

---

[17]There are approaches to identity that would accept the "temporary identity" and "temporary non-identity" of Socrates and β. While I believe that neither such an approach to

So something has to go: either [always, anti-symmetry] or [always, strong supplementation]. I suggest rejecting the former, and letting the structural requirement of anti-symmetry express itself in another way. Instead of saying that it is always true that any mutual parts are identical, it will say, roughly, that any things that are always mutual parts are identical. (The exact expression of the axiom has to be slightly more nuanced to get the desired effect, given our treatment of $\leq$ and existence.) Call this "anti-always-symmetry."

What do we lose with anti-always-symmetry instead of [always, anti-symmetry]? One of the key formal features of Classical Mereology is that, in it, the $\leq$ relation is "extensional" in a couple senses. Here are two theorems of CM:

(e1) $\forall x \forall y (\forall z (z \leq x \leftrightarrow z \leq y) \rightarrow x = y)$
(e2) $\forall x \forall y (\forall z (z \circ x \leftrightarrow z \circ y) \rightarrow x = y)$.

(e1) is a consequence of the conjunction of reflexivity and anti-symmetry. (e2) is a consequence of strong supplementation and anti-symmetry. Both theorems answer to what might be taken to be a component of the core conception: no difference among objects unless there is a $\leq$-difference (as in (e1)), and, in particular, a difference of a certain substantial character (as in (e2)). Different objects must have different spreads. My suggestion is that we get something importantly answering to these classical ideas in (T) if we have such theorems as these:

(e1′) $\forall x \forall y (\ \text{Always}[\forall z (z \leq x \leftrightarrow z \leq y)] \rightarrow x = y)$
(e2′) $\forall x \forall y (\ \text{Always}[\forall z (z \circ x \leftrightarrow z \circ y)] \rightarrow x = y)$.

Thus we retain the idea that different objects must somehow have non-trivial differences in "spread;" but we allow that that difference may show up only in their past or future spreads. This accords nicely, I suggest, with the idea that a fusion for a condition is something that is *always* spread across reality in a way that matches the condition. Further, it gives us a version of flatness: that no non-empty condition has two distinct diachronic fusions. For (e2′) guarantees that if x and y always have the same spread, then $x = y$. Thus we lose classical extensionality, but we retain an important echo of it.[18] And, crucially, we have that every not-always-empty condition has at most one diachronic fusion.

I admit that it is not obvious that my suggested candidates for expressions of the structural ideas of anti-symmetry and strong supplementation, namely anti-always-symmetry and [always, strong supplementation], are superior to any other candidates that might be found. But here are some reasons in support of my candidates, *given* my suggestion about fusion. Consider again the diachronic fusion β from a few paragraphs back. While Socrates is sitting, Socrates $\leq \beta$, and every part of β overlaps Socrates (and vice-versa). My suggestions have it that $\beta \leq$ Socrates. By contrast, suppose we are committed to [always, anti-symmetry], and therefore reject [always, strong supplementation]. What, then, should we think is "making" it the case that the "arrow" points from Socrates to

identity nor its rejection is essential to the package I have been calling "the tensed approach to time," I take such approaches to be highly problematic, so I set them aside.

[18]This fact also might be used to help answer questions like "What grounds the differences in features between two objects that currently have exactly the same parts?" See Wasserman [30] for a discussion of how a parameterist view that has much in common with our TM1 should answer such questions, and for some reasons of a different sort to prefer rejecting anti-symmetry while holding on to [always, strong supplementation].

β and not vice-versa? In this particular case, we might see that β is a "hokey" fusion while Socrates is a "natural" fusion. (It is almost trivial that Socrates is a fusion of "$x =$ Socrates." Everything is a fusion of *some* condition, so the contrast isn't between "fusions" and "natural" objects.) But there is more: this theory would have it that, in general, whenever two distinct objects have the same overlappers, either neither is part of the other, or the arrow points one way only. Sometimes there appears to be a free "option." E.g., take any fusion of the condition on x "$x \leq$ Socrates and $x \neq$ Socrates." It has the same overlappers as Socrates, but this theory gives us nothing to go on as to whether Socrates is part of it, vice-versa, or neither. Similarly, if γ_1 "conjunctively fuses" Socrates and Plato (i.e., fuses the condition on x "Socrates and Plato exist and x is one of them"), γ_2 conjunctively fuses Socrates and Aristophanes, and γ_3 conjunctively fuses Aristophanes and Plato, what shall we say about the part-whole relations among: conjunctive fusions of any two of the γ_i; of all three of them; of Socrates and γ_3; of the three men; of the parts of the three men, etc.? Many different patterns of answer are compatible with this theory. My suggestion, on the other hand, yields clear answers in all such cases, by [always, strong supplementation]. My proposal also has the advantages of retaining clear expressions of what are plausibly core ideas about fusions like "everything is the fusion of its parts," as well as the echoes of classical extensionality just discussed.

2.1.3 Flatness

A further reason for following my suggestion is that it nicely implements the idea that composition is flat, as well as plenitudinous. (But, as we have seen, on my suggestion, composition is mandated to be flat only with respect to *diachronic* fusion; synchronic fusion can easily fail to be flat.) Moreover, if we adopt the axioms that [always, to be or to have a part requires existence], that [always, parthood is transitive], and that [always, each not-always-empty condition has a *unique* diachronic fusion], the resulting axiom set is equivalent with the axiom set thus far proposed.

2.2 Formalization of first candidate

I will now give my first candidate for an expression of the core, in one formalization of the (T) treatment of time. The language is $\mathcal{L}^T(\leq)$, which is a first-order modal language with identity, the $\leq$ relation, and two one-place sentential operators $\mathbf{P}$ and $\mathbf{F}$ meant to correspond to "it was (at some time) the case that" and "it will be (at some time) the case that." Define $\mathbf{A}\phi$ (meant to correspond to "it is always the case that") as $\neg\mathbf{P}\neg\phi \wedge \phi \wedge \neg\mathbf{F}\neg\phi$. Similarly, define $\mathbf{S}\phi$ ("it is at sometime the case that") as $\phi \vee \mathbf{P}\phi \vee \mathbf{F}\phi$.

I will assume that we are working with a logic for the temporal operators on which $\mathbf{A}$ and $\mathbf{S}$ behave just as $\square$ and $\lozenge$ do in variable-domain quantified S5. (Again, in much of the following, we may provisionally address modal mereology by replacing $\mathbf{A}$ and $\mathbf{S}$ with $\square$ and $\lozenge$.) In particular, I will assume we are working in a variable-domain quantified tense logic that is sound and complete for models with non-empty domains (i.e., at least one world has at least one object in its domain, though empty worlds are allowed) and whose frames are in the class

of frames in which: the "earlier than" relation on the worlds is irreflexive (no world is earlier than itself), anti-symmetric (no distinct worlds are each earlier than the other), linear (one of any pair of distinct worlds is earlier than the other), transitive, and infinite on both sides in the sense that for each world, there is at least one earlier world, and at least one later world. (We will not here take up the project of modifying our expression of flat plenitude in ways that are appropriate for other tense logics, e.g., ones with branching or circular time.)

Such a quantified tense logic can be cooked up basically by combining the tense logic $\mathbf{L}_4$ of [2] with the quantified modal logic, with existence predicate, of the first half of chapter 16 of [14] appropriately combined with a standard identity theory, as in chapter 17 of [14]. We do not need a primitive existence predicate, however, given the identity theory. We use the defined existence predicate E so that Ex is just

$$\exists y \; y = x.$$

It is a theorem that everything exists ($\forall x \mathrm{E} x$), and it is also a theorem that everything is always self-identical ($\forall x \mathbf{A} x = x$). It is not, of course, a theorem that everything always exists ($\forall x \mathbf{A} \mathrm{E} x$).

Now my proposed "structural" axioms are these four:

(Ex) $\mathbf{A} \forall x \mathbf{A} \forall y \, \mathbf{A}(x \leq y \to (\mathrm{E} x \wedge \mathrm{E} y))$
(ATr) $\mathbf{A} \, \forall x \forall y \forall z (x \leq y \wedge y \leq z \to x \leq z)$
(ASSUP) $\mathbf{A} \, \forall x \forall y \, (\forall z(z \circ x \to z \circ y) \to x \leq y)$
(WeakAS) $\mathbf{A} \, \forall x \mathbf{A} \forall y (\, (\mathbf{A}(\mathrm{E} x \to x \leq y) \wedge \mathbf{A}(\mathrm{E} y \to y \leq x)) \to x = y).$

Note that (ASSUP) implies

(ARef) $\mathbf{A} \forall x \, x \leq x.$

And (ASSUP) and (Ex) together yield $\mathbf{A} \forall x \, \mathbf{A}(\mathrm{E} x \leftrightarrow x \leq x)$.

Now to state our one further axiom (scheme), we will use some definitions. Define "synchronic fusion" schematically as follows: let "$SFu(b, [x|\phi_x])$" abbreviate

$$\forall x(\phi_x \to x \leq b) \wedge \forall y(y \leq b \to \exists x(\phi_x \wedge y \circ x))$$

(where y is not free in ϕ_x). This definition is, of course, the same as the definition of fusion we used for CM1 (except that now ϕ_x may be drawn from the larger language $\mathcal{L}^T(\leq)$).

Now there is a slightly tricky point about synchronic fusing, as defined and in the presence of the structural axiom (Ex): if, at some time, a condition ϕ_x is empty (with respect to x), i.e., $\neg \exists x \phi_x$, then, at that time, every non-existent object synchronically fuses ϕ_x. That is to say, the "modal closure" of every instance of the following is a theorem:

$$\neg \exists x \phi_x \wedge \neg \mathrm{E} b \to SFu(b, [x|\phi_x]).$$

(Here, and in the following, take the "modal closure" of ψ to be the wff

$$\mathbf{A} \forall v_1 \mathbf{A} \forall v_2 \ldots \mathbf{A} \forall v_n \, \mathbf{A} \psi$$

where $v_1, \ldots v_n$ are the free variables of ψ, in order of appearance.) For, if nothing is ϕ and b does not exist, by (Ex), nothing is part of b, and so both

clauses of the synchronic fusion definition here are vacuously satisfied. Moreover, with (ARef) in place as well, we have that only non-existent things can ever synchronically fuse a (then) empty condition:

$$\neg \exists x \phi_x \wedge SFu(b, [x|\phi_x]) \rightarrow \neg \mathrm{E}b.$$

For, if $SFu(b, [x|\phi_x])$ and b exists, then, by (ARef) $b \leq b$ and so b overlaps something that is ϕ; but there are no such things, by hypothesis. Similarly, a non-existent thing cannot (while it fails to exist) synchronically fuse a non-empty condition, since if $\exists x \phi_x$ and $SFu(b, [x|\phi_x])$, then $\exists x \, x \leq b$.

One might dislike this way of talking and require instead that for b to synchronically fuse a condition requires, in addition to what we have stated, that b exist. But the current notion of synchronic fusion not only duplicates the classical definition of fusion, it has the benefit of giving us a nice characterization of our desired notion of "diachronic fusion." We may define it thus: let "$DFu(b, [x|\phi_x])$" abbreviate

A $SFu(b, [x|\phi_x])$.

Then we get as corollaries to the above results about synchronic fusion that anything that ever diachronically fuses a condition, in general, exists when and only when something meets that condition. That is, we have (the modal closure of every instance of)

$$DFu(b, [x|\phi_x]) \rightarrow \mathbf{A}(\exists x \phi_x \leftrightarrow \mathrm{E}b).$$

And so nothing that ever exists can ever diachronically fuse a condition that is never satisfied. So it is always true that every always-empty condition has no diachronic fusion. And even though a thing synchronically fuses every then empty condition at a time at which it does not exist, it will not (then) diachronically fuse every then empty condition.

My current candidate for the fusion existence axiom scheme, then, is this: for any wff ϕ_x, the modal closure of

(DFE) $\mathbf{S} \exists x \phi_x \rightarrow \mathbf{S} \exists b \, DFu(b, [x|\phi_x])$

is an axiom (with suitable constraints to avoid variable collisions).

I will label this system TM1 (first candidate for tensed mereology).

2.2.1   A few theorems

We now look at a few theorems, to help give a feel for this formalism and its relation to CM. Our first result is that TM1 is equivalent with the axiom set TM1′, consisting of (Ex), (ATr), and the following scheme: (the modal closure of any instance of)

(DFUE) $\mathbf{S} \exists x \phi_x \rightarrow \mathbf{S} \exists! b \, DFu(b, [x|\phi_x])$.

The equivalence is not completely obvious. To show that every instance of (DFUE) is derivable in TM1 is not too difficult (see below), but the other direction seems to require a little more effort. We confine a sketch of an argument for it to a footnote.[19]

---

[19]To get the result, it is helpful first to derive, within TM1′, (ARef) and (WeakAS) modifying the derivation outlined in footnote 22 of [13]. Begin by deriving, as a very useful

Another basic result about TM1 is that something very close to CM comes out true, indeed always true. Define $x \simeq y$ as $x \leq y \wedge y \leq x$. Now, roughly put, if we "count by the equivalence relation $\simeq$" rather than "count by identity," the traditional CM axiomatization comes out true at each time. Obviously we will have transitivity and strong supplementation, and it is easy to show that at each time, if something is then ϕ, there is then a synchronic fusion of ϕ. The other bit we get is that there is "at most one" synchronic fusion of ϕ, where we "count by $\simeq$." That is to say,

$$\exists x \phi_x \rightarrow \forall b \forall k (SFu(b, [x, \phi_x]) \wedge SFu(k, [x|\phi_x]) \rightarrow b \simeq k).$$

This can be shown with (ASSUP) together with (ATr). Of course we do not get anti-symmetry, (but we do get the triviality that $x \leq y \wedge y \leq x \rightarrow x \simeq y$).

There is a predicate closely related to $\simeq$ that we will be especially interested in later: the two-place predicate $x \equiv_\circ y$ ("x and y have the same existing overlappers"), defined as

$$\forall z(z \circ x \leftrightarrow z \circ y).$$

In TM1, we have as a theorem the universal closure of

$$x \equiv_\circ y \leftrightarrow x \simeq y$$

but we do not get its modal closure, since $x \equiv_\circ y$ holds when x and y both do not exist. We do have the modal closure of

$$\mathbf{A}\, x \equiv_\circ y \; \leftrightarrow \; x = y,$$

and we cannot replace $\equiv_\circ$ with $\simeq$ here, since the right-to-left direction can fail in the result.

Diachronic fusions are unique in TM1: if a condition ever has a diachronic fusion then it never has two. To see this, note that for each condition ϕ_x we may deduce (the modal closure of)

$$DFu(b, [x|\phi_x]) \wedge DFu(c, [x|\phi_x]) \rightarrow \mathbf{A}\, b \equiv_\circ c.$$

Thus we may justifiably speak of *the* (diachronic) fusion of a condition. A more general result is that "co-intensive" conditions have the same fusions. Letting $\mathcal{C}[\phi]$ denote the modal closure of ϕ, we have every instance of

$$\mathcal{C}[\phi \leftrightarrow \psi] \rightarrow \\ \mathcal{C}[DFu(b, [x|\phi_x]) \wedge DFu(c, [x|\psi_x]) \rightarrow \mathbf{A}\, b \equiv_\circ c]$$

where the variables b and c are not free in ϕ or ψ.

---

lemma, (the modal closure of) $Ex \rightarrow x \circ x$. Then, to derive (ASSUP), first show that for any wff ψ, if $\mathbf{S}(Ea \wedge \psi)$ then we have that, at any time: $DFu(b, [x|x = a \wedge \psi]) \wedge Eb$ only if $b \leq a$. (To show this: if $\mathbf{A}(Ea \rightarrow \psi)$ then $DFu(a, [x|x = a \wedge \psi])$; if not, get c with $DFu(c, [x|x = a \wedge \neg \psi])$ and e with $DFu(e, [x|x = b \vee x = c])$. Show that $DFu(e, [x|x = a])$ and conclude $a = e$. From this it is easy to deduce that $b \leq a$.) Then, temporarily abbreviate "$\forall z(z \circ x \rightarrow z \circ y)$" as χ and suppose that at some time $Ex \wedge Ey \wedge \chi$ holds. Get objects b and c with $DFu(b, [w|(w = x \vee w = y) \wedge \chi])$ and $DFu(c, [w|w = y \wedge \chi])$. By the previous result, Ec yields $c \leq y$. Now show that $DFu(b, [w|w = y \wedge \chi])$, conclude $b = c$, and so $\mathbf{A}((Ex \wedge Ey \wedge \chi) \rightarrow (x \leq b \wedge b \leq y))$. (ASSUP) is easy from there.

2.2.2 Proper parts and atoms

Given the failure of classical anti-symmetry, the notions of "proper part" and "mereological atom" need some adjustment.

Classically, x is called a "proper part" of y when $x \leq y$ yet $x \neq y$. But in TM1, this could happen when $x \simeq y$, and we will want to distinguish this relation from a more "robust" proper part relation. Accordingly, let us say that x is an "asymmetric part" of y when $x \leq y \wedge y \not\leq x$. Writing $x \ll y$ for this, we even have the echo of a key theorem of CM (sometimes used as an axiom), namely weak supplementation. Our theorem is:

(WS′) $\forall x \forall y (x \ll y \rightarrow \exists z (z \wr x \wedge z \leq y))$

(where, again, $x \wr y$ means $\neg\, x \circ y$). This is not the classical weak supplementation theorem, since we defined $\ll$ as asymmetric part rather than as "proper part" classically conceived. (Within CM, the two definitions are equivalent, but not within TM1.)

Classically, a mereological atom is something that is its own sole part—or, equivalently, has no classical proper parts. But in TM1, the corresponding notion should be the notion of something that has no asymmetric part. Say that x is "temporarily atomic" when $Ex \wedge \neg \exists y\, y \ll x$. Say that x is "permanently atomic" when it is always temporarily atomic, i.e., $\mathbf{A} \neg \exists y\, y \ll x$. In general, temporary atomicity does not imply permanent atomicity. But every temporarily atomic thing has at least one permanently atomic part, for, if y is temporarily atomic at some time, there is then a b such that $DFu(b, [x | x = y \wedge y$ is temporarily atomic]$)$, and this b is permanently atomic.

Say that something is "temporarily gunky" if it exists and has no temporarily atomic parts. We may say that the universe is "temporarily atomistic" if everything has a temporarily atomic part, or, equivalently, nothing is temporarily gunky. Then we can prove in TM1 that if the universe is temporarily atomistic, then (at that time)

$\forall x \forall y (x \simeq y \leftrightarrow \forall z (z$ is temporarily atomic $\rightarrow (z \leq x \leftrightarrow z \leq y)))$.

This echoes a classical theorem to the effect that in an atomistic world, objects are the same iff they have the same atomic parts. Say that the universe is "permanently atomistic" if it is always temporarily atomistic. In such a universe, we get further echoes: the modal closures of

$DFu(b, [x | x$ is temporarily atomic $\wedge x \leq y]) \leftrightarrow b = y$
$DFu(b, [x | x$ is permanently atomic $\wedge x \leq y]) \leftrightarrow b = y$

(a thing is the fusion of its atomic parts, in either sense). Accordingly, objects can be individuated by their atoms, in a sense: we get the modal closure of

$x = y \leftrightarrow \mathbf{A}((Ex \leftrightarrow Ey) \wedge$
$\phantom{x = y \leftrightarrow \mathbf{A}((}\forall z (z$ is temporarily atomic $\rightarrow (z \leq x \leftrightarrow z \leq y)))$.

2.2.3 Temporal parts

Our fusion axiom scheme "generates" both "big" objects (e.g., the diachronic fusion of the condition "x is either Socrates or Plato") and "small" objects (e.g.,

the diachronic fusion of the condition "x is Socrates and x is sitting").[20] In fact, it generates some maximally temporally small objects, (at least, in one sense of that phrase) for: if any object y exists at two times that are different in the sense that some proposition P is true at one and false at the other, then we have the fusion of "$[x|x = y \land P]$," which will $\simeq y$ whenever it exists, but will not exist at both times. And if P is true only at one time, a time at which y exists, then the fusion of "$[x|x = y \land P]$" will answer to many, if not all, of the properties a "stage", "time-slice," or "instantaneous temporal part" is supposed to have.[21]

Is this a form of perdurance? Only in a fairly weak sense. Consider this classic statement of the distinction between perdurance and endurance, from David Lewis:

> Let us say that something *persists* iff, somehow or other, it exists at various times; this is the neutral word. Something *perdures* iff it persists by having different temporal parts, or stages, at different times, though no one part of it is wholly present at more than one time; whereas it *endures* iff it persists by being wholly present at more than one time. Perdurance corresponds to the way a road persists through space; part of it is here and part of it is there, and no part is wholly present at two different places. Endurance corresponds to the way a universal, if there are such things, would be wholly present wherever and whenever it is instantiated. Endurance involves overlap: the content of two different times has the enduring thing as a common part. Perdurance does not. ([17] p. 202.)

Note that Lewis' explanation of perdurance seems to involve a notion of being (existing) *at* different times *by* having (as parts) stages that are (exist) *at* those times.[22] It is worth pointing out that it is no part of TM1 that y exists at the two times *in virtue of* having two stages, one at each time. TM1 says nothing about "in virtue of" connections, and it is perfectly possible to add to it the claim that for some object y, every "temporal restriction" of it exists in virtue of y, or in virtue of y's having certain features; it is also possible to add to it a claim going the other way around. Similar themes are usefully discussed and elaborated by Hawthorne in [12].

Further, the closure of $Ey \to (x \leq y \to Ex)$ is a theorem of TM1, following easily from (Ex), so: whenever y exists, anything that is part of it also exists. In this sense, y is "wholly present" whenever y exists. (More on this in our discussion of perdurance vs. endurance below.) Further, if we think of the "content of a time" as being the fusion of the things that exist at that time, then y may easily be part of two times in the sense that it was part of the earlier time, and

---

[20] Of course, these conditions involve predicates that are not in $\mathcal{L}^T(\leq)$; we are imagining applying the scheme to an enlargement of our pure language.

[21] There are issues here regarding the possibility of worlds in which, intuitively, time passes, and yet there are no such P's (e.g., because of the eternal recurrence of some span of history). Our devices for simulating talk of times may not work as desired for such worlds.

[22] This is not to burden Lewis with the doctrine that a perduring thing exists *simpliciter* in virtue of (the existences of) its parts, nor with the doctrine that stages *are more basic than* wholes of which they are parts. We can respect his explicit disavowal of the latter thesis in Postscript B of "Survival and Identity" in [16]. The point concerns being-at-t, or existing-at-t, not plain existing.

became part of the later; we thus have a kind of overlap of times, which Lewis seems to take to be a hallmark of endurance. (But we do not have a timeless "has" on which both times timelessly have y as a part; nor is it true at any one time that both times have y as a part. So we do not exactly match what Lewis' words strictly seem to demand.) (We get the very same result if we think of the "content of a time" as the fusion of the things that exist at *and only at* that time; see below.)

Given that the temporal vocabulary in our language is built up from the past and future operators, we have not built in the resources to talk directly about the individual "times" that would be used in the standard possible-worlds style semantics for our language.[23] But let us suppose that we joined our theory with an object language "time theory." Suppose we had some further axioms (and perhaps a larger language) that somehow gave us the following resources: a (not necessarily primitive) one-place object-language predicate $Time$, with (the modal closures of) the following as theorems of time theory

$Time(x) \rightarrow \mathbf{A}\, Time(x)$
$\exists x\, Time(x)$.

In addition, time theory should provide a one place predicate $Clock$, corresponding to "the time on the clock is," with the modal closures of the following as theorems

$\exists x\, Clock(x)$
$(Clock(x) \wedge Clock(y)) \rightarrow (Time(x) \wedge x = y)$
$Time(x) \rightarrow \mathbf{S}\, Clock(x)$.

Thus, always, there is a unique time on the clock, and every time is, at some time, on the clock. Further, we want the clock always to be changing; we want the modal closure of

$Clock(x) \rightarrow (\neg\, \mathbf{P} Clock(x) \wedge \neg\, \mathbf{F} Clock(x))$.

Now, in any theory that includes a time theory and TM1 together with any instances of the fusion scheme that may be built up with any new predicates we introduce to build time theory, we may deduce that, in a certain sense, everything has a temporal part, and each thing is the diachronic fusion of its temporal parts. Define $Inst(x)$ ("x is instantaneous") as

$\mathbf{S}(\mathrm{E}x \wedge \neg\, \mathbf{P}\mathrm{E}x \wedge \neg\, \mathbf{F}\mathrm{E}x)$.

Now define $Slice(y,x)$ ("y is (currently) a time-slice of x") as

$Inst(y) \wedge \mathrm{E}x \wedge \mathrm{E}y \wedge x \equiv_\circ y$.

These definitions are stated in our basic language $\mathcal{L}^T(\leq)$.

Now in our enlarged theory, we have that everything has a unique slice whenever it exists, and that everything is the diachronic fusion of its slices. Formally, the modal closures of the following are theorems:

$\mathrm{E}z \rightarrow \exists! y\, Slice(y,z)$
$DFu(b, [x|Slice(x,z)]) \leftrightarrow b = z$.

---

[23] The use of "time theory" in this section to simulate expressive resources like outer quantifiers and "now" is like the use of certain assumptions about possibility that allow actualists to simulate possible worlds and possible individuals, as in Fine [10].

The first is obtained with time theory and the instance of (DFE) on the condition $[x|x = z \wedge Clock(t)]$, and the second is straightforward.

We can get a time theory within $\mathcal{L}^T(\leq)$ by adding to TM1 the axiom

(INST) $\mathbf{A}\exists x\, Inst(x)$.

For we may then define $Time(x)$ as

$\mathbf{S}(\mathbf{E}x \wedge Inst(x) \wedge \forall y\, y \leq x)$

and $Clock(x)$ as

$Time(x) \wedge \mathbf{E}x$.

All the needed theorems can be derived. To get the existence of times, note that we can deduce the theorem

$\forall z(Inst(z) \rightarrow \exists b\, DFu(b, [x|\mathbf{E}z \wedge x = x]))$.

Times are then time-slices of the universe, so to speak.[24] It is worth noting that we would get the very same result if we defined a time, alternatively, as a sometime existing and instantaneous thing that then has every *instantaneous* thing as part of it. For if t is a time in the above, "wider" sense, then it is obviously a time in the alternative "narrower" sense, and if t is a time in the narrow sense then, for any y, whenever t exists, if y exists, then y has a slice (the fusion of $[x|x = y \wedge \mathbf{E}t]$) that is part of t and of which y is a part; hence y is part of t. So the two definitions are equivalent in TM1.

Putting these results together, we see that if we add a time theory to TM1, even if we use some new primitive predicates to do so, we get a time theory that is statable within $\mathcal{L}^T(\leq)$ alone. Hence we may as well take the addition of time theory to TM1 to simply be the addition of (INST) to TM1.

We will make heavy use of an interesting effect of time theory. Time theory allows us to simulate the "possibilist" or "outer" quantifiers Σ (for some) and Π (for any), intended to range over all past, present, and future objects. For suppose we wish to say that some future individual will be a philosopher wiser than any today:

$\Sigma z(z$ will be a philosopher and z is wiser than any existing philosopher$)$.

(Read "a is wiser than b" to mean basically "a, at the height of a's powers, has more wisdom than b at the height of b's.") As is well known, using the resources of our two tense operators, unaided, we will have trouble.[25] But given that we have time theory at our disposal, we get the desired result with

$\exists t(Clock(t) \wedge \mathbf{F}\exists z(z$ is a philosopher $\wedge\ \mathbf{A}(Clock(t) \rightarrow \forall x(x$ is a philosopher $\rightarrow z$ is wiser than $x))))$.

In general, we can render $\Sigma x\, \psi$ and $\Pi x\, \psi$ as, respectively,

---

[24]It is also worth considering weaker theories that arise from axioms short of (INST), like one saying that it is always the case that if something exists, then something instantaneous exists. This allows for times when nothing exists, which (INST) does not. But it does not, in general, allow us to simulate the outer quantifiers.

[25]For we do not mean what we get if we replace Σ with $\exists$ in the above; that implies that there is now someone who is wiser than any current philosopher. Rather, we mean

$\mathbf{F}\exists z(z$ is a philosopher and NOW $\forall x(x$ is a philosopher $\rightarrow z$ is wiser than $x))$.

But this requires a "NOW" operator; deleting it in the above yields something we do not intend.

$$\exists t(Clock(t) \wedge \mathbf{S}\exists x\mathbf{A}(Clock(t) \to \psi))$$
$$\exists t(Clock(t) \wedge \mathbf{A}\forall x\mathbf{A}(Clock(t) \to \psi))$$

(where t is not free in ψ). Similarly, we can simulate devices of temporal cross-reference like "then," "at that time," "at some later time," etc., and cumulative quantifiers like "there have been."

2.3   Second Candidate

As we noted, even without a time theory, we get some things that look something like "temporal parts" in TM1, and the addition of a time theory gives us things that look like "instantaneous" temporal parts. Yet reflection on how one conceives of the difference between events and objects (in a thick sense of "object," on which it is not trivial that everything is an object) in time suggests that TM1 gives us nothing but objects. The contrast I have in mind is this: objects are typically thought to be "wholly present" at a time, while events are "spread out" in time. Again, we have (the modal closure of):

$$x \leq y \to \mathrm{E}x$$

which tells us that anything that ever exists is ever a part of a given thing only when the putative part exists. So, in a very strong sense, everything is "wholly present" whenever it exists: whenever it exists, *all* (quantifying over past, present and future objects) of its (then) parts (then) exist.

It is natural to think of events differently. Echoing Lewis' gloss of perdurance, we tend to think of a current event as having earlier and later parts in much the same way as the way a road has parts ahead of us and behind us. Earlier events, even ones that are over, and hence don't currently exist, are even now parts of an ongoing protracted event. The eternalist, who regards holdings of the part-whole relation to be a timeless affair, has a fairly natural way of rendering these thoughts, but how should they be rendered in the tensed conception?

My suggestion is that we allow for the event to have earlier and later parts in the following way: an event, unlike an object, can have parts that no longer exist, and parts that are yet to exist. Further, the mereological structure of the class of events is forever unchanging. That is, in general, if x is an event-part of y (i.e., an event that is part of y), then it is always the case that x is part of y. We might allow, however, that events can pick up and lose non-events as parts. Among events, what changes is only which event-parts of the event exist. Picturesquely, as time passes from before to after the war: the war first does not exist, and none of its event-parts exist. Then it comes into existence (begins), and, for a few years, exists and always has an existing instantaneous part and many existing "segments." During these years it also has earlier event-parts (that then no longer exist) and later event-parts (that are then yet to exist). Finally, it is over: the war no longer exists, and accordingly, neither do any of its event-parts. It is then true to say that all of its event-parts existed, but do not exist, and never will exist again; nevertheless, they remain parts of it. (If all this sounds unnatural, try replacing "exists" with "occurs" throughout.)

One other plausible difference in our conceptions of events and things is that events "perdure," or exist at different times *in virtue of* having temporal parts

that are "proper to" those times, or exist only at those times. Without worrying about exactly what "in virtue of" amounts to here, we may take this idea to require, at least, that an event has, at each time at which it exists (is occurring), an event that is part of it, but only exists at that time.

We can now see that the typical four-dimensionalist package includes both *mereological eternalism* and *slicing*. The former insists that parthood is primarily an atemporal or eternal affair: x is part of y (or not) once and for all, or, more accurately, "outside of time." The latter says that anything that is spread out in time has parts that are temporally minimally spread out. The two are separable; we have already seen that TM1 with time theory yields slicing, but is certainly not eternalist. (Hawthorne employed something like the parameterist flat plenitude sketched at the outset to make basically this point.) As we will see, we may also separate out the *eternalist* attitude while retaining the *permanence* of holdings and failings to hold of parthood: one can take tense seriously, while holding that parthood is temporally rigid, and that everything perdures.

2.3.1   Endurance and perdurance

Returning to our main thread, and joining our discussion of perdurance with our previous idea that, among events, parthood is permanent, we get the result that an instantaneous event is always part of itself, even when it does not exist.

For the rest of the paper, we will use the terms "endurant" and "perdurant," according to the conceptions of object and event, respectively, just outlined. Let us say that a thing x "endures" if nothing ever is a non-existent part of x. ("Thing" will be a neutral word, so that both endurants and perdurants are things; similarly, quantifiers with "thing" in them, "something," "everything," and so on are neutral.) It is worth noting that this definition of endurance in terms of parthood (and temporal operators) corresponds nicely with the intuitive notion of an endurant as something that is "wholly present at each time at which it exists."[26] Being present is existing, and being wholly present is like being wholly red: every part is present. Again, in TM1, everything endures.

Let us say that a thing x "perdures" if x does not endure; i.e., if at some time there is something y that at some time both fails to exist and is part of x. TM2 will allow for perdurers. Note that an instantaneous object (an object that exists only at one time) may endure or perdure (but not both).

It should be openly admitted that some aspects of what follows are driven by metaphysical considerations that may be exterior to the core mereological idea of flat plenitude. For there may be other possible formal theories that are even more neutral than TM2 and still are not completely unreasonable as candidates for counting as expressions of the core within (T), for example, our TM3. This reveals that TM1 was more metaphysically committed than it initially might have seemed.

---

[26]This might be touted as an advantage of the perspective afforded by taking tense seriously. Giving a good non-trivial gloss for the notion of "wholly present" is known to be tricky; see Thomas Crisp and Donald Smith's [8] for a discussion of the difficulties and a possible solution to them. The gloss given here would not satisfy Crisp and Smith, unfortunately. See also Sider's discussion in 3.3 of [25], in which Sider writes (on p. 64) that "no one would deny that a part of an object at a given time must exist then."

And it should be mentioned, at least briefly, that there are rather different perspectives on the mereological differences between objects and events; a leading alternative would have it that there are two fundamentally different relations of parthood (and two corresponding modes of composition) for the two different kinds of things. Kit Fine argues for a distinction of this sort in [9]. (Even if there are two fundamentally different relations, one might wonder whether there are nonetheless some formal principles governing their disjunction; our TM2 might be entertained as such. As briefly discussed below, TM2 could be augmented in some straightforward ways to yield what would, in effect, be a mereology with two disjoint sub-networks of things, one for the endurants, one for the perdurants.)

2.4   Formalization of second candidate

We will use the following abbreviations:

$\mathcal{E}(x)$ for $\mathbf{A}\forall y\mathbf{A}(y \le x \to \mathrm{E}y)$
$\mathcal{P}(x)$ for $\neg\mathcal{E}(x)$.

$\mathcal{E}(x)$ says that x endures. Note that x perdures ($\mathcal{P}(x)$) just in case: at some time, there is something that at some time is part of x and does not exist. We have the modal closure of

$(\mathcal{E}(x) \lor \mathcal{P}(x)) \land \neg(\mathcal{E}(x) \land \mathcal{P}(x))$

as a logical truth. And we have $\mathcal{E}(x) \to \mathbf{A}\mathcal{E}(x)$, and similarly for $\mathcal{P}$.

The first two purely "structural" axioms of TM2 are the modal closures of these:

(MCTr) $x \le y \land y \le z \; \to \; x \le z$
(WeakAS2) $(\mathbf{A}(x \le x \to x \le y) \land \mathbf{A}(y \le y \to y \le x)) \; \to \; x = y$.

Modally closed transitivity (MCTr) says that transitivity holds among any three things, no matter when they exist. (It implies (ATr) from TM1, but not vice-versa. Yet it is a theorem of TM1, following by temporal logic alone from (Ex) and (ATr).) (WeakAS2), our current version of anti-symmetry, is a theorem of TM1, and could have been substituted for (WeakAS) in axiomatizing TM1, given (Ex).

Our third axiom is a version of strong supplementation. Its force is most perspicuously revealed if it is stated using the "outer" quantifiers. Let $x \bullet y$ abbreviate

$\Sigma z(z \le x \land z \le y)$.

Our third axiom would be the modal closure of

(PSSUP$^\Sigma$) $\mathrm{E}x \land \mathrm{E}y \to (\Pi z(z \bullet x \to z \bullet y) \to x \le y)$

if we allowed the possibilist quantifiers in our language. Recall that time theory would give us the ability to simulate those quantifiers. Officially, we do not so allow, and our axiom (PSSUP) will be cast in $\mathcal{L}^T(\le)$. (PSSUP) will be something that, in the presence of the other axioms, has the force of (PSSUP$^\Sigma$); we will indicate how this is done later.

Next we have two axioms concerning reflexivity: the modal closures of

(Refl1) $\mathrm{E}x \to x \le x$

$$\text{(Refl2)} \quad x \le y \to (x \le x \land y \le y).$$

In fact, (Refl1) is redundant, being derivable from (PSSUP). (Refl2) might be motivated as follows: though we allow endurants to fail to be parts of themselves (in fact we require that they are not parts of themselves when they do not exist) things cannot "enter the mereological web" unless they are parts of themselves. Hence, a non-existing endurant is not part of anything (existent or otherwise), nor does it have parts.

Next, an axiom concerning existence and parthood: the modal closure of

$$\text{(Ex2)} \quad \mathrm{E}x \land x \le y \ \to \mathrm{E}y.$$

This is a weakening of (Ex). We don't allow an existing thing to be part of a non-existent thing.

Next, two axioms governing the behavior of perdurants. They are the modal closures of:

$$\text{(Perdurance)} \quad \mathcal{P}(x) \to \mathbf{A}(\mathrm{E}x \to \exists y(\mathcal{P}(y) \land Slice(y,x)))$$
$$\text{(PRigid)} \quad \mathcal{P}(x) \land \mathcal{P}(y) \to (x \le y \to \mathbf{A}x \le y).$$

(Perdurance) basically says that perdurants must have perduring slices. (PRigid) says that parthood is rigid, among perdurants.

Note that from (PRigid) and (Refl1) we may deduce the modal closure of

$$\mathcal{P}(x) \to \mathbf{A}x \le x.$$

This marks a distinctive feature of perdurants, for us: each always has itself as a part (even when it does not exist). Note that from (Refl2), (Ex2) and (PRigid), we may deduce the closure of this as a theorem:

$$x \le y \to \mathbf{S}(\mathrm{E}x \land \mathrm{E}y \land x \le y).$$

(It is part of our logic that there is never anything that never exists.) Endurants, by definition, never have non-existent parts, and, as we saw, by (Refl2) a non-existent endurant cannot be part of something else. And though a perdurant may both have perduring parts and be a part of other perdurants when it does not exist, all such parthood connections must be "witnessed" at a time when both of the connected things exist. Hence, an instantaneous perdurant does not, when it exists, have any non-existing parts.

Finally, we have our two fusion axiom schemes for TM2: one for endurants, one for perdurants. These schemes require enduring fusions for conditions that are satisfied only by endurants, and similarly with perdurants. The need for separating out the two schemes, and for their restricted and conditional nature, will be explained further in the discussion under the heading "Mixture" below. The first scheme: the modal closure of every instance of

$$\text{(CEFE)} \quad \mathbf{S}\exists x(\mathcal{E}(x) \land \phi_x) \to$$
$$\mathbf{S}\exists b(\mathcal{E}(b) \land DFu(b, [x|\mathcal{E}(x) \land \phi_x])).$$

is an axiom.

Second, given that parthood is rigid among perdurants, we need a significant restriction on the existence of perduring fusions. Let us use "$Rigid(\phi)$" to abbreviate the modal closure of

$$\phi \to \mathbf{A}\phi.$$

Then our axiom scheme is the modal closure of

(CPFE) $(Rigid(\phi_x) \wedge \mathbf{S}\exists x(\mathcal{P}(x) \wedge \phi_x)) \to$
$\mathbf{S}\exists b(\mathcal{P}(b) \wedge DFu(b, [x|\mathcal{P}(x) \wedge \phi_x])).$

TM2, then, is the above set of axioms (modulo our promise to cast (PSSUP$^\Sigma$) in $\mathcal{L}^T(\leq)$).

Since we have $\mathbf{S}\exists x\, x = x$ as a truth of our temporal logic, there must at some time be either an endurant or a perdurant. Hence at least one of the two fusion schemes of TM2 will be "activated."

Before proceeding, let us note that we have the following result: suppose that we have that at some time

$$DFu(b, [x|\phi_x]) \wedge DFu(c, [x|\phi_x]).$$

We may derive that if b and c are both endurants, or are both perdurants, then $b = c$. But, it could be that one is an endurant and the other a perdurant. So the uniqueness of fusions may have to be qualified, if we allow both endurants and perdurants.

2.4.1 Pure perdurance

It is consistent with TM2 that there are never any perdurants. In fact, if we add a sentence to that effect to TM2, the resulting theory is identical with the theory of TM1.

It is consistent with TM2 that there are never any endurants. Let us use "PurePer" for the result of adding to TM2 the statement that there are never any endurants. The resulting theory corresponds nicely with the four-dimensionalist's (F) picture of mereology.

In (F), we might say that parthood relations are "eternal," (outside of time) since parthood is a two-place relation (there is no temporal parameter) and there is no tense. In PurePer, parthood relations are "sempiternal," (in time, but the same at all times) since there are never any endurants and parthood on perdurants is rigid: if ever x is part of y, then always; so if ever x isn't part of y, then never is x part of y.

More generally, PurePer gives us a sempiternal mereology that is, in a way, a "moving image" of the (F) theorist's eternal mereology. Moving (think of the moving spotlight), yet mereologically unchanging. *What exists* may change from time to time, but the part-whole connections to not change. Now, the (F) theorist's mereology is just plain old CM. Consider the language $\mathcal{L}^T(\Sigma, \leq)$ which is $\mathcal{L}^T(\leq)$ augmented with "outer" quantifiers Σ and Π. (Think of the smaller language as literally a subset of the larger.) For any wff ϕ of $\mathcal{L}^T(\leq)$, let ϕ^* in $\mathcal{L}^T(\Sigma, \leq)$ be the result of replacing, in ϕ, every occurrence of $\exists$ with Σ and every occurrence of $\forall$ with Π. Finally, for every ϕ in $\mathcal{L}^T(\Sigma, \leq)$, let $\tau(\phi)$ be its "translation," using the method outlined above, into $\mathcal{L}^T(\leq)$, with the background time theory provided by PurePer, and let ϕ_Π be $\tau(\phi^*)$. Then we have the following result: for any closed wff of $\mathcal{L}(\leq)$ (again, the smaller language being thought of as a subset of the larger language)

ϕ is a theorem of CM1 iff $\mathbf{A}\,\phi_\Pi$ is a theorem of PurePer.[27]

Thus, the tenser can render, in (T), with PurePer, an image of the four-dimensionalist's mereology, and without recourse to an apparatus of spatio-temporal regions and occupation relations. Where in (F) we would make heavy use of the notion of occupying a region of space-time, together with some kind of mereology on the regions themselves, all we need in (T) is the notion of existence—essentially bound up with the difference between the $\exists$ quantifier and the Σ quantifier—along with our temporal operators. In (F), the notion of occupation is needed in order to make non-trivial sense even of the idea of a thing existing at one time but not others; in (T) this idea is basically given as part of the fundamental apparatus.[28]

2.4.2   Strong supplementation

As promised, we now give our axiom (PSSUP) in $\mathcal{L}^T(\leq)$. Recall that (PSSUP$^\Sigma$) is

$$(\text{PSSUP}^\Sigma) \quad \mathrm{E}x \wedge \mathrm{E}y \to (\Pi z(z \bullet x \to z \bullet y) \to x \leq y)$$

We first note that at any time at which there exist no perdurants, we need not "worry" about non-existent things (since then we may assume x and y are endurants, and so no things that then fail to exist are then parts of either), and so the standard supplementation formulation, with its standard (inner) quantifiers, does what we need. And at a time when there exists at least one perdurant, b, b has a slice s that exists only at this time, and this allows us to simulate the outer quantifiers locally (at this time). So our axiom, then, is

$$(\text{PSSUP}) \quad \mathbf{A}(\neg\,\exists x\,\mathcal{P}(x) \to \forall x \forall y\,(\forall z(z \circ x \to z \circ y) \to x \leq y)) \wedge$$
$$\mathbf{A}\forall s((\mathcal{P}(s) \wedge Inst(s)) \to \forall x \forall y($$
$$[\mathbf{A}\forall z(\ \mathbf{S}\exists w \mathbf{A}(\mathrm{E}s \to (w \leq z \wedge w \leq x)) \to$$
$$\mathbf{S}\exists w \mathbf{A}(\mathrm{E}s \to (w \leq z \wedge w \leq y)))]$$
$$\to x \leq y)).$$

We might wish to entertain a slightly stronger axiom, one that would be expressed with the outer quantifiers as the modal closure of

---

[27]This can be proved by establishing correspondences between models of CM1 and PurePer and appealing to the completeness of each of the respective logics. For any model $\mathfrak{M}$ of CM1, we can construct a model $\mathfrak{N}$ of PurePer in which, basically, for any wff $\phi \in \mathcal{L}(\leq)$, ϕ is true in $\mathfrak{M}$ just in case ϕ_Π is true at every world in $\mathfrak{N}$. The simplest way to do this is, roughly, to put all the things from the domain of $\mathfrak{M}$ into one world's domain, and one only, and to interpret $\leq$, at every world in $\mathfrak{N}$, the same as in $\mathfrak{M}$. (There are other, structurally more interesting, corresponding models which differ on which things exist at which times.) Going the other way is at least as easy.

[28]Tangentially, we briefly note that somewhat similarly, a four-dimensionalist might create an image of an endurantist's metaphysics within (F), by creatively weakening classical assumptions about the occupation relation and the connections between mereology and occupation. Now, in one way, the tenser gets by with less: (T) does not require an ontology of space-time and a fundamental occupation relation to express the relevant claims: $\leq$ is the only non-logical piece of ideology. But, of course, (T) achieves this only by use of its temporal operators, hence one might chalk up a roughly balancing point for (F): though it needs more ontology and another relation, it needs no "logical" notions beyond first-order logic. Thus it is unclear, just here, which package (operators or space-time with occupation) is more "parsimonious." This sort of situation is not unfamiliar in contemporary metaphysics.

$\quad$ (PSSUP2$^\Sigma$) $\quad x \leq x \land y \leq y \to (\Pi z(z \bullet x \to z \bullet y) \to x \leq y)$.

The stronger axiom is only necessary if we allow perdurants and endurants to "mix" mereologically, as discussed immediately below. There are difficulties expressing (PSSUP2$^\Sigma$) without making primitive use of the outer quantifiers or adding (INST) as an axiom. Nonetheless, we can achieve its effect with the modal closure of

$\quad$ (EPPart) $\quad (\mathcal{P}(x) \land \mathcal{E}(y) \land y \leq x) \to \exists z(\mathcal{P}(z) \land z \leq y)$

for EPPart, in conjunction with the other axioms of TM2, guarantees that (PSSUP2$^\Sigma$) holds. (Proving this is a little non-trivial; make repeated use of the observation that we are now guaranteed the holding of the modal closure of $(x \bullet y \land \mathcal{P}(x) \land \mathcal{P}(y)) \to \Sigma z(\mathcal{P}(z) \land z \leq x \land z \leq y)$.)

2.4.3 Mixture

TM2 is consistent with the co-existence of perdurants and endurants. The resulting structure is a bit less clean than that determined by either of the "pure" theories TM1 or PurePer. The inclusion of (EPPart) seems a reasonable supplement to TM2, but even with it, much is up for grabs.

Say that x and y have the same "parity" if they are both endurants or both perdurants. It is consistent with TM2 that if x and y have different parities, then x is never part of y, and one can consider adding an axiom to that effect: (the modal closure of)

$\quad$ (Sep) $\quad x \leq y \to (\mathcal{E}(x) \leftrightarrow \mathcal{E}(y))$.

(Sep) removes the need for (EPPart), for it also guarantees (PSSUP2$^\Sigma$). But it is consistent with TM2 that things with different parities are sometimes related by the part relation.

TM2 requires that if $x \leq y$ and $\mathcal{P}(x)$ and $\mathcal{E}(y)$, then $Inst(x)$.[29] But TM2 is much more tolerant of endurants as parts of perdurants, and, at the other extreme from (Sep), we could generate many such things by adding an axiom stating that endurants have perduring slices. Consider

$\quad$ (PSlice) $\quad \mathbf{A}\forall x(\mathcal{E}(x) \to \exists y(\mathcal{P}(y) \land Slice(y, x)))$.

By (PSSUP), an endurant with a perduring slice is (just for that moment) a part of that slice, and hence, by (MCTr) is then a part of any temporally extended perdurant the slice is a part of. So (PSlice) yields many cases of endurants that are parts of perdurants. One can show that if (PSlice) is added to TM2, then, the modal closure of every instance of

$\quad$ (GCPFE) $\quad (Rigid(\phi_x) \land \mathbf{S}\exists x\phi_x) \to$
$\qquad\qquad\qquad\qquad \mathbf{S}\exists b(\mathcal{P}(b) \land DFu(b, [x|\phi_x]))$

is a theorem.[30] It is worth pointing out that the condition of rigidity is necessary,

---

[29]If at one time $\mathcal{P}(x) \land \mathcal{E}(y) \land x \leq y$, then if x exists at some other time, at the other time it has, by (Perdurance), a perduring slice z that is, by (PRigid), a part of it back at the first time; but then, by (MCTr), z is part of y at the first time while z does not exist; but endurants can't have non-existing parts.

[30]Use (CPFE) to get b with $\mathcal{P}(b) \land DFu(b, [y|\mathcal{P}(y) \land (\exists x(\mathcal{P}(x) \land \phi_x \land x = y) \lor \exists x(\mathcal{E}(x) \land \phi_x \land Slice(y, x)))])$, and confirm that $DFu(b, [x|\phi_x])$.

for the structural axioms of TM2 rule out the existence of fusions for certain non-empty conditions. For example, one can deduce (without appealing to the fusion axioms) that

$$(\mathcal{P}(x) \wedge \mathcal{P}(y) \wedge \mathbf{A}x \wr y \wedge \mathbf{S}(\mathrm{E}x \wedge \mathrm{E}y \wedge \psi) \wedge \mathbf{S}(\mathrm{E}x \wedge \mathrm{E}y \wedge \neg \psi)) \rightarrow$$
$$\neg \mathbf{S} \exists b \, DFu(b, [z|(z = x \wedge \psi) \vee (z = y \wedge \neg \psi)]).$$

This, despite the fact that the condition in question is sometimes satisfied; hence not every instance of the (DFE) scheme from TM1 is a theorem of TM2. This is a kind of limit on plenitude in TM2: it is left open that there might be a condition that is sometimes satisfied yet which has no diachronic fusion.

To get more of a feel for what happens when we add (PSlice) and tolerate a mixture of endurants and perdurants, we take, for example, Socrates. Supposing that he endures, there is a perduring fusion of $[x|x = \text{Socrates}]$; call it "Socrates' life." At every time at which Socrates exists, Socrates' life exists, and vice-versa. And if we confine our eyes to existing things, the two are mereologically quite similar: at each such time, Socrates $\equiv_\circ$ his life. Yet it is not the case that Socrates $\simeq$ his life: he is part of it, but it is not part of him. They differ in a couple other ways: first, Socrates never has any non-existing parts, while Socrates' life always has as parts non-existent past and future slices (and segments, e.g., Socrates' childhood, the perduring fusion of $[x|\mathbf{S}(x$ is a perduring slice of Socrates and Socrates is a child$)]$). Second, his life will always be part of itself, and will always possess the same perduring parts, even when it ceases to exist. Socrates, on the other hand, has no parts whatsoever after he ceases to exist, and, again, never has non-existent perdurants as parts. Further, if Socrates exists at noon, then he has both a perduring slice and an enduring slice at noon.[31] Both these things exist only at noon, and both are, at noon, $\equiv_\circ$ with Socrates, and indeed $\simeq$ with Socrates. The perduring slice is always part of itself, and always part of Socrates' life, but it immediately ceases to be a part of Socrates. The enduring slice immediately ceases to be part of itself, or anything else. In fact, at any time at which Socrates exists, the only perdurants that are then part of Socrates are those that are parts of his perduring slice at that time.

Of course there appears to be a certain amount of artificiality here. As suggested earlier, the choices made to get TM2 reflect certain metaphysical ideas that are arguably exterior to the core issues of mereology. And there are other natural ideas. For example, one might accept the basic characterization of endurants and perdurants given here, but weaken the fusion axioms of TM2 in various ways. One might keep (CPFE) so that the formation of perduring fusions is completely unrestricted, while simply deleting (CEFE), so that there are no guarantees about enduring fusions. One could then have it that there are both endurants and perdurants, with the mereological structure of the perdurants being a plenitudinous, completely regular, and well-structured sub-network of the total mereology, but with a "sparse" and formally "unpredictable" mereo-logical sub-network of endurants. (E.g., one could have it that only living things endure, or that only fundamental particles endure, or, in a version of van In-wagen's view in [28], only fundamental particles and living things endure, and so forth.) Each endurant would have a corresponding perduring image (as with Socrates and his life, above) and one could even have an axiom to the effect

---

[31]The enduring slice is the enduring fusion of $[x|x = Socrates \wedge Clock(n)]$.

that the endurants function as basic, in that all the perdurants can be "built up" by slicing and fusing the perduring images of endurants. It would perhaps capture this idea to say that always, every perdurant overlaps a perduring slice of an endurant.

2.5   Third candidate

Our third candidate for tensed mereology, TM3, will essentially involve the "outer" quantifiers. Accordingly, we will take (INST) as an axiom. (Of course, if the outer quantifiers were admitted as primitive, we could drop (INST) and get a theory TM3$'$ which would not logically entail the existence of instantaneous things.) From a purely mathematical point of view, TM3 seems to be a leading candidate for expressing flat plenitude within the tensed perspective. One might say, very roughly speaking, that it yields (or makes room for) all of the things yielded by all of the other theories considered, and still more (though the notion of yielding "the same things" is somewhat elusive in this context). At the same time, it imposes a nice structure: for every condition, there is exactly one thing that fuses it (individuating conditions intensionally).

The guiding idea of TM3 is basically that the distinction between the existent and the non-existent is completely ignored by the mereology. For example, now to fuse the condition "x is at some time a cat" requires now having as a part every (outer quantifier) thing that is at some time a cat. But it is not stipulated one way or the other whether the fuser itself exist. Existence is taken to be, as it were, external to mereological considerations, allowed to vary completely independently from mereological facts. But it is natural to work with a quantified tense logic in which there is (outer quantifier) nothing that never exists, and it is easy to imagine supplementing TM3 with axioms connecting parthood and existence; Ex2 is a natural candidate.

As a first attempt, we would seem basically to want to ensure general transitivity with (MCTr) and to add a "unique fusion" axiom like

   (DFUE$^\Pi$?) $\mathbf{S}\,\Sigma x \phi_x \;\rightarrow \mathbf{S}\,\Sigma! b \; DFu(b, [x|\phi_x])$

while redefining DFu using Σ and Π in place of $\exists$ and $\forall$. Yet a technical quirk arises here, and its simplest solution ultimately seems to be to include "null" things, things that are parts of everything. The quirk can be brought out by considering fusions of conditions that are, at a given time, empty.

Consider a diachronic fusion, call it α, of the condition "x is a sitting cat" at a time at which there are (outer quantifier) no sitting cats. (Since we will be wanting the outer quantifiers throughout most of this discussion, let us briefly signify their use by adding a superscript Π or Σ to English quantifier words.) According to TM1, such a fusion both fails to exist and fails to be part of itself, when there exist no sitting cats; *a fortiori* when there are$^\Sigma$ no sitting cats. According to TM2, we might have an event-like fusion of this condition, which would remain part of itself at this time, while not existing. For TM3, we seem to need to choose arbitrarily whether, in general, a fusion of a then-empty condition should be then part of itself. But whichever way we go, we run into difficulties.

If we say that α is then part of itself, we will need to amend our definition of fusion just considered. For when there are$^\Sigma$ no sitting cats, this definition has it that, since α is part of itself, for some$^\Sigma$ x, x is a sitting cat and $\alpha \bullet x$; contradiction. (Please forgive the use/mention subtleties we are ignoring here and in the next paragraph.)

If, instead, we say that α then is not part of itself, we run into another difficulty. For suppose there is some condition ϕ_x that α always uniquely$^\Pi$ satisfies (even when it does not exist).[32] Now given this supposition, if β fuses ϕ_x, then, when α is not part of itself, still α satisfies ϕ_x, so, is part of β, and hence $\alpha \bullet \alpha$; so α has to have at least one$^\Sigma$ part, even though it is not part of itself. This appears to be an unmotivated awkwardness, and there are further difficulties down the road that we will not detail here.

The smoothest solution, mathematically speaking, seems to be to re-model our tensed mereology on Boolean algebra rather than on CM: to require "zero" or "null" elements, things that are parts of everything.[33] At any time, a fusion for a then-empty condition is then a zero. We define our notion of fusion on the model of a minimal upper bound. First, let $SFu^\Pi(b, [x|\phi_x])$ abbreviate

$$\Pi x(\phi_x \to x \le b) \wedge \Pi y(\Pi x(\phi_x \to x \le y) \to b \le y)$$

(where y is not free in ϕ_x). Note that if ϕ_x is empty$^\Sigma$ at a time, then, for any$^\Pi$ thing, every$^\Pi$ ϕ_x is part of it, and hence anything that then fuses ϕ_x is part of every$^\Pi$ thing, i.e., is a zero.

Now, define $DFu^\Pi(b, [x|\phi_x])$ to abbreviate

$$\mathbf{A} SFu^\Pi(b, [x|\phi_x])$$

and we can state our fusion axiom scheme for TM3: for every ϕ_x, the modal closure of

$$(\text{DFE}^\Pi) \quad \Sigma b\, DFu^\Pi(b, [x|\phi_x])$$

is to be an axiom.

Our analog of anti-symmetry will be the closure of

$$(\text{WeakAS}^\Pi) \quad (\mathbf{A} x \le y \wedge \mathbf{A} y \le x) \to x = y.$$

It turns out that to get the desired structural effects, including an analog of strong supplementation, in this context (especially given our definition of fusion), it is not inappropriate to use a less familiar sort of axiom. Let $x \restriction y$ abbreviate

$$\Pi z(z \le x \wedge z \le y \to \Pi w(z \le w))$$

so that $x \restriction y$ just when the only$^\Pi$ common parts of x and y are zeroes. Then our axiom, the closure of

$$(\text{AComp}^\Pi) \quad \Sigma y(x \restriction y \wedge \Pi z((z \restriction x \to z \le y) \wedge (z \restriction y \to z \le x)))$$

basically says that everything always has at least one$^\Sigma$ "complement," a thing

---

[32]$x = \alpha$ is traditionally taken to be such a condition, a tradition we have followed, though one might consider the idea of having non-existent things fail to be self-identical. Unless we go for a fairly serious departure from classical logic, the problem would still arise, however, e.g., for the condition $\mathbf{S} x = \alpha$.

[33]For a discussion of the connections between CM and Boolean algebra see [22] and [13].

that is, as it were, maximally disjoint from it.

TM3 then, is the axiom set consisting of (INST), (MCTr), (WeakAS$^\Pi$), (AComp$^\Pi$), and every instance of (DFE$^\Pi$).

We may note that (AComp$^\Pi$) by itself entails

$$\mathbf{A}\Pi x\, x \leq x$$

and, with (ATr$^\Pi$), entails a variant strong supplementation principle, geared to the Boolean-algebra-like setting. The variant is the modal closure of

$$\Pi z(z \restriction y \rightarrow z \restriction x) \rightarrow x \leq y.$$

Another main structural principle that is a theorem of TM3, one that shows an important connection between the defined notion DFu^Π and the other definitions of mereological sum is (the closure of):

$$DFu^\Pi(b, [x|\phi_x]) \rightarrow \Pi y(\Pi x(\phi_x \rightarrow y \restriction x) \rightarrow y \restriction b).$$

In general, the formal situation is not unlike that with TM1; modulo the $\simeq$ relation, we have, at each time, a Boolean algebra. And we clearly have a kind of flatness for our plenitude, for, as is fairly easy to derive, (the closure of) every instance of

$$DFu^\Pi(b, [x|\phi_x]) \wedge DFu^\Pi(c, [x|\phi_x]) \rightarrow b = c$$

is a theorem.

A model (using the standard possible-worlds semantics) for TM3 can be constructed as follows: let the worlds and accessibility relation be given by a copy of the positive and negative integers under the standard less-than ordering, calling the set of worlds Z. For the individuals, let E be some non-empty set, let J be the power set of E, and H be the set of all functions f with domain Z and range J. H is to be the set of individuals for the model. For each world $w \in Z$, let its domain (the set of individuals that "exist at" w) be the set of all $f \in H$ such that $f(w)$ is non-empty, together with the "always empty" function, i.e., the f whose value at every integer is the empty set. This way, every member of H "exists at" at least one world. Now interpret $\leq$ in the model as follows: for each world w, the extension of $\leq$ at w is to be the set of pairs $\langle x, y \rangle$ such that $x(w) \subseteq y(w)$. Thus, for each world w, the function f such that $f(w) = E$ and $f(v) = \emptyset$ for $v \in Z$ with $v \neq w$ is, effectively, an instantaneous time-slice of the universe: at w, it exists, and everything is part of it, and it exists at no other times. Picturesquely, we have an image of a universe with "sempiternal atoms" (the constant functions whose values are always singletons) that exist at all times, together with their slices and an always-existing always-null thing, and everything that "can be" fused out of these ingredients, where, aside from the permanently null thing, a thing exists just when it has a non-null part.

Null things may seem to be metaphysically unmotivated. But consider intuitively the fusions for the conditions $[x|x$ is a living dinosaur$]$ and $[x|x$ is a living brontosaurus$]$. Intuitively, these fusions should not currently have any existing parts. But is each currently part of itself? Is each currently part of the universe? Is the latter fusion currently part of the former? Our intuitions don't seem to be especially strong here, though perhaps TM1's answers (three "no"s) are favored, at least for the first two questions. TM3 settles all these questions by saying that the fusions for these conditions are currently parts of everything$^\Pi$ (even

the non-existent things), which might be regarded as a convention about how to settle awkward cases of part-whole relations, about on a par with the convention that everything is part of itself. To help loosen resistance, one might point out that currently, for every thing x, indeed for every$^\Pi$ x, it is in fact true that every (indeed every$^\Pi$) living brontosaurus, and every living dinosaur, is part of x! So why shouldn't these fusions be? The two fusions (and their distinctness) are easy to motivate by considerations about the past; what is hard to settle is their present mereological behavior. The always-null thing is harder to motivate than the temporarily null things, since it *never* has any parts beside itself, and thus seems never to have any "positive existence." It is straightforward, however, to tweak TM3 so as to eliminate the always-null thing, but leave everything else the same.

2.5.1   Conclusion

We have seen that a plausible expression of CM's core idea of flat plenitude, in a tensed setting, should make use of some notion like our notion of diachronic fusion, so that the main ideas would be some strong version of transitivity and that every suitable condition has exactly one diachronic fusion. Beyond that, there are choices and tweaks to be made, and though we have narrowed the field significantly, it is not obvious which ones are best.[34]

3   Appendix: summary of definitions and axiom sets

Formal language: $\mathcal{L}^T(\leq)$, a first-order modal language with identity, the $\leq$ relation, and two one-place sentential operators $\mathbf{P}$ and $\mathbf{F}$ meant to correspond to "it was (at some time) the case that" and "it will be (at some time) the case that." $\mathbf{A}\phi$ is defined as $\neg\mathbf{P}\neg\phi \wedge \phi \wedge \neg\mathbf{F}\neg\phi$, and $\mathbf{S}\phi$ is defined as $\phi \vee \mathbf{P}\phi \vee \mathbf{F}\phi$. $\mathrm{E}x$ is defined as $\exists y \; y = x$. $x \circ y$ is defined as $\exists z(z \leq x \wedge z \leq y)$. Let "$SFu(b, [x|\phi_x])$" abbreviate

$\quad \forall x(\phi_x \to x \leq b) \wedge \forall y(y \leq b \to \exists x(\phi_x \wedge y \circ x))$

(where y is not free in ϕ_x) and let "$DFu(b, [x|\phi_x])$" abbreviate

$\quad \mathbf{A} \; SFu(b, [x|\phi_x]).$

First Candidate: TM1

$\quad$ (Ex) $\mathbf{A}\forall x\mathbf{A}\forall y \, \mathbf{A}(x \leq y \to (\mathrm{E}x \wedge \mathrm{E}y))$
$\quad$ (ATr) $\mathbf{A}\,\forall x\forall y\forall z(x \leq y \wedge y \leq z \to x \leq z)$
$\quad$ (ASSUP) $\mathbf{A}\,\forall x\forall y\,(\forall z(z \circ x \to z \circ y) \to x \leq y)$
$\quad$ (WeakAS) $\mathbf{A}\,\forall x\mathbf{A}\forall y(\,(\mathbf{A}(\mathrm{E}x \to x \leq y) \wedge \mathbf{A}(\mathrm{E}y \to y \leq x)) \to x = y)$
$\quad$ (DFE) $\mathbf{S}\,\exists x\phi_x \; \to \mathbf{S}\,\exists b \; DFu(b, [x|\phi_x])$

(DFE) is an axiom scheme, and the intention is that *the modal closure* of every instance of it is an axiom. The modal closure of wff ψ (sometimes denoted $\mathcal{C}[\psi]$) is

$$\mathbf{A}\forall v_1 \mathbf{A}\forall v_2 \ldots \mathbf{A}\forall v_n \ \mathbf{A}\psi$$

where $v_1, \ldots v_n$ are the free variables of ψ, in order of appearance.

Axiom set TM1$'$ (equivalent with TM1)

(Ex), (ATr), and the following scheme:

$\quad$ (DFUE) $\quad \mathbf{S}\,\exists x \phi_x \ \rightarrow \mathbf{S}\,\exists! b \ DFu(b, [x|\phi_x])$.

More definitions

$\quad x \simeq y$ is defined as $x \leq y \wedge y \leq x$.
$\quad x \equiv_\circ y$ is defined as $\forall z(z \circ x \leftrightarrow z \circ y)$.
$\quad Inst(x)$ is defined as $\mathbf{S}(\mathrm{E}x \wedge \neg\,\mathbf{PE}x \wedge \neg\,\mathbf{FE}x)$.
$\quad Slice(y, x)$ is defined as $Inst(y) \wedge \mathrm{E}x \wedge \mathrm{E}y \wedge x \equiv_\circ y$.
$\quad Time(x)$ is defined as $\mathbf{S}(\mathrm{E}x \wedge Inst(x) \wedge \forall y\, y \leq x)$.
$\quad Clock(x)$ is defined as $Time(x) \wedge \mathrm{E}x$.

An important potential axiom or theorem considered is

$\quad$ (INST) $\quad \mathbf{A}\exists x \ Inst(x)$.

Assuming (INST) holds, to simulate the "outer" quantifiers Σ and Π, we define $\Sigma x\,\psi$ and $\Pi x\,\psi$ as, respectively,

$\quad \exists t(Clock(t) \wedge \mathbf{S}\exists x \mathbf{A}(Clock(t) \rightarrow \psi))$
$\quad \exists t(Clock(t) \wedge \mathbf{A}\forall x \mathbf{A}(Clock(t) \rightarrow \psi))$

(where t is not free in ψ).

$\quad \mathcal{E}(x)$ is defined as $\mathbf{A}\forall y \mathbf{A}(y \leq x \rightarrow \mathrm{E}y)$.
$\quad \mathcal{P}(x)$ is defined as $\neg\,\mathcal{E}(x)$.

$Rigid(\phi)$ is defined as $\mathcal{C}[\phi \rightarrow \mathbf{A}\phi]$.

Candidate two: TM2

The modal closures of

$\quad$ (MCTr) $\quad x \leq y \wedge y \leq z \ \rightarrow \ x \leq z$
$\quad$ (WeakAS2) $\quad (\mathbf{A}(x \leq x \rightarrow x \leq y) \wedge \mathbf{A}(y \leq y \rightarrow y \leq x)) \ \rightarrow \ x = y$
$\quad$ (PSSUP) $\quad \mathbf{A}(\neg\,\exists x\,\mathcal{P}(x) \rightarrow \forall x \forall y\,(\forall z(z \circ x \rightarrow z \circ y) \ \rightarrow \ x \leq y)) \wedge$
$\qquad\quad \mathbf{A}\forall s((\mathcal{P}(s) \wedge Inst(s)) \rightarrow \forall x \forall y(\\$
$\qquad\qquad [\mathbf{A}\forall z(\ \mathbf{S}\exists w \mathbf{A}(\mathrm{E}s \rightarrow (w \leq z \wedge w \leq x)) \rightarrow\\$
$\qquad\qquad\qquad \mathbf{S}\exists w \mathbf{A}(\mathrm{E}s \rightarrow (w \leq z \wedge w \leq y)))] \\$
$\qquad\qquad \rightarrow x \leq y))$
$\quad$ (Refl1) $\quad \mathrm{E}x \rightarrow x \leq x$
$\quad$ (Refl2) $\quad x \leq y \rightarrow (x \leq x \wedge y \leq y)$
$\quad$ (Ex2) $\quad \mathrm{E}x \wedge x \leq y \ \rightarrow \mathrm{E}y$

(Perdurance) $\mathcal{P}(x) \rightarrow \mathbf{A}(Ex \rightarrow \exists y(\mathcal{P}(y) \wedge Slice(y,x)))$
(PRigid) $\mathcal{P}(x) \wedge \mathcal{P}(y) \rightarrow (x \leq y \rightarrow \mathbf{A}x \leq y)$
(CEFE) $\mathbf{S}\exists x(\mathcal{E}(x) \wedge \phi_x) \rightarrow$
$\qquad\qquad \mathbf{S}\exists b(\mathcal{E}(b) \wedge DFu(b, [x|\mathcal{E}(x) \wedge \phi_x]))$
(CPFE) $(Rigid(\phi_x) \wedge \mathbf{S}\exists x(\mathcal{P}(x) \wedge \phi_x)) \rightarrow$
$\qquad\qquad \mathbf{S}\exists b(\mathcal{P}(b) \wedge DFu(b, [x|\mathcal{P}(x) \wedge \phi_x])).$

The axiom (PSSUP) has the force of

$\qquad$ (PSSUP$^\Sigma$) $Ex \wedge Ey \rightarrow (\Pi z(z \bullet x \rightarrow z \bullet y) \rightarrow x \leq y)$

without requiring the assumption of (INST) as an axiom (where $x \bullet y$ is defined as $\Sigma z(z \leq x \wedge z \leq y)$).

Third candidate: TM3

(See main text.)

References

[1] Elizabeth Barnes and Robert Williams. A theory of metaphysical indeterminacy. In Karen Bennett and Dean W. Zimmerman, editors, *Oxford Studies in Metaphysics*, volume 6. Oxford University Press, 2011.

[2] John Burgess. Basic tense logic. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic 7*, volume 165, pages 1–42. Kluwer, 2002.

[3] Michael B. Burke. Dion and theon: An essentialist solution to an ancient puzzle. *Journal of Philosophy*, 91(3):129–139, 1994.

[4] Michael B. Burke. Preserving the principle of one object to a place: A novel account of the relations among objects, sorts, sortals, and persistence conditions. *Philosophy and Phenomenological Research*, 54(3):591–624, 1994.

[5] Rudolf Carnap. *Meaning and Necessity: A Study in Semantics and Modal Logic*. University of Chicago Press, Chicago, 1947.

[6] Roderick Chisholm. Parts as essential to their wholes. *Review of Metaphysics*, 26(4):581–603, 1973.

[7] Roderick Chisholm. *Person and Object: A Metaphysical Study*. Open Court, 1976.

[8] Thomas M. Crisp and Donald P. Smith. 'wholly present' defined. *Philosophy and Phenomenological Research*, 71(2):318–344, 2005.

[9] Kit Fine. Compounds and aggregates. *Noûs*, 28(2):137–158, 1994.

[10] Kit Fine. The problem of possibilia. In Michael J. Loux and Dean W. Zimmerman, editors, *The Oxford Handbook of Metaphysics*, pages 161–179. Oxford University Press, 2003.

[11] Kit Fine. Tense and reality. In *Modality and Tense*. Oxford University Press, 2005.

[12] John Hawthorne. Three-dimensionalism. In *Metaphysical Essays*. Oxford University Press, 2006.

[13] Paul Hovda. What is classical mereology? *Journal of Philosophical Logic*, 38(1):55–82, 2009.

[14] G.E. Hughes and M.J. Cresswell. *A New Introduction to Modal Logic*. Routledge, 1996.

[15] Mark Johnston. Is there a problem about persistence? *Aristotelian Society, Supplementary Volume*, 61:107–35, 1987.

[16] David Lewis. *Philosophical Papers*, volume I. Oxford University Press, 1983.

[17] David Lewis. *On the Plurality of Worlds*. Basil Blackwell, Oxford, 1986.

[18] David Lewis. *Parts of Classes*. Blackwell, 1991.

[19] David Lewis. Tensing the copula. *Mind*, 111(441):1–14, 2002.

[20] Vann McGee. How we learn mathematical language. *Philosophical Review*, 106(1):35–68, 1997.

[21] D.H. Mellor. *Real Time II*. Routledge, 1998.

[22] Carsten Pontow and Rainer Schubert. A mathematical analysis of theories of parthood. *Data and Knowledge Engineering*, 59:107–138, 2006.

[23] W. V. O. Quine. *Word and Object*. MIT Press, Cambridge, MA, 1960.

[24] Michael C. Rea. Constitution and kind membership. *Philosophical Studies*, 97(2):169–193, 2000.

[25] Theodore Sider. *Four-Dimensionalism*. Oxford, 2001.

[26] Alfred Tarski. Foundations of the geometry of solids. In John Corcoran, editor, *Logic, Semantics, Meta-Mathematics*. Hackett, 1983.

[27] Judith Jarvis Thomson. The statue and the clay. *Noûs*, 32(2):149–173, 1998.

[28] Peter van Inwagen. *Material Beings*. Cornell University Press, 1990.

[29] Peter van Inwagen. Can mereological sums change their parts? *The Journal of Philosophy*, 103(12):614–630, 2006.

[30] Ryan Wasserman. The standard objection to the standard account. *Philosophical Studies*, 111(3), 2002.

[31] Ryan Wasserman. The problem of change. *Philosophy Compass*, 1(1):48–57, 2006.

[32] Byeong-Uk Yi. The logic and meaning of plurals I. *Journal of Philosophical Logic*, 34(5–6):459–506, October 2005.

[33] Byeong-Uk Yi. The logic and meaning of plurals II. *Journal of Philosophical Logic*, 35(3), June 2006.